Economic Principles of Space Traffic Control

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Abstract

Open access to Earth's orbits presents a unique regulatory challenge. In this paper, I derive economic principles governing the choice of space traffic control policies. I show that policies which target satellite ownership, such as satellite taxes or permits, achieve greater expected social welfare than policies which target satellite launches, such as launch taxes or permits. Price or quantity policies can achieve equal expected social welfare due to the symmetry of uncertainty between regulators and firms. I also show that active debris removal can reduce the risk of runaway debris growth no matter how it is financed, but can only reduce the risk of satellite-destroying collisions if satellite owners pay for it. Technical solutions to space traffic control tend to emphasize launch restrictions or public funding of debris removal technology development and use, but often ignore that current and prospective orbit users dissipate rents under open access. While satellite-focused policies can achieve first-best orbit use, attempts to control orbital debris growth and collision risk through launch fees or debris removal subsidies under open access may be ineffective or backfire.

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1 Introduction

Open access to common-pool resources tends to cause resource overuse or stock collapse (Gordon, 1954; Hardin, 1968; Dietz, Ostrom, and Stern, 2003). Despite awareness of this fact for over one hundred years, new and existing common-pool resources are still plagued by open access problems (Stavins, 2011). Open access orbit use has led to the accumulation of orbital debris, from nonoperational satellites to nuts, bolts, and propellant fuel particulates. Collisions between orbiting bodies can shatter satellites into thousands of dangerous high-velocity fragments, some of which may be too small to track. Runaway debris growth, known as Kessler Syndrome, threatens to render high-value orbits unusable for decades or centuries (Kessler and Cour-Palais, 1978). As technology makes satellites cheaper to launch and more reliable, firms are planning to launch thousands of satellites into already-congested orbits. The need for policies to manage orbital congestion is more pressing than ever. While commons management problems have been studied extensively (Weitzman, 1974; Ostrom, 1999; Newell and Pizer, 2003; Costello, Gaines, and Lynham, 2008), engineers, economists, and policymakers know little about how space traffic should be managed and debris removal technologies should be employed. In this paper I answer two fundamental questions of space traffic control. First, what do optimal space traffic control policies look like? Second, how should active debris removal be employed? The key insights of my paper are that space traffic control policies should target satellites in orbit rather than satellite launches, and that satellite owners must pay for debris removal for it to reduce equilibrium collision risk.

I derive economic principles of space traffic control policy in the first dynamic model of satellite launch and ownership with physical uncertainty over collisions and positive feedbacks in debris growth. I highlight the key policy design constraints imposed by open access and show how the use of active debris removal technologies will affect equilibrium collision risk and debris growth. I show that despite uncertainty over the risk of catastrophic collisions, the traditional "prices vs. quantities" question is moot. Price or quantity policies can achieve first-best outcomes because both regulators and firms are equally uncertain about the collision risk. The key design issue is whether the regulator's policy targets satellites in orbit (for example, a satellite tax) or the act of launching satellites (for example, a launch tax).[1] In the setting I study, regulating satellites in orbit achieves higher expected social welfare than regulating the act of launching satellites. Regulating satellite launches instead of satellites in orbit creates rents to satellite ownership and induces suboptimal spikes in equilibrium collision risk just before the policy takes effect. Satellite launch controls are also limited in their ability to induce deorbits, and optimal satellite launch controls have unfavorable dynamic properties.

[1]While I study principles relevant to orbital regulation, I do not explicitly analyze the problem of a global orbit use regulator tasked with creating an efficient and self-enforcing international agreement on orbit use. Such analysis is eminently important to the problem of orbit use, but beyond my scope in this paper.

Contrary to predictions from non-economic models of orbit use, active debris removal may reduce the debris stock without affecting equilibrium collision risk. If satellite owners receive debris removal for free, more launchers will enter to take advantage of the cleared space. For active debris removal to reduce equilibrium collision risk, satellite owners must bear the cost of removal.

Prior analyses have quantified the costs and benefits of mitigating and reducing debris and collision risk (Liou and Johnson, 2008, 2009b; Bradley and Wein, 2009; Ansdell, 2010; Schaub et al., 2015; Macauley, 2015), noted that open access and the common-pool nature of orbits make rational actors ignore their effects on other orbit users (Merges and Reynolds, 2010; Weeden and Chow, 2012; Adilov, Alexander, and Cunningham, 2015; Salter, 2015; Rao and Rondina, 2018), and considered necessary legal and institutional features that an orbit use management policy framework ought to have (Weeden, 2010; Weeden and Chow, 2012; Akers, 2012).[2] I build on these analyses by formally modeling open access incentives with a realistic dynamic structure which reveals feedbacks between the environment and orbit-users. This structure allows me to identify issues with launch taxes and publicly-provided debris removal not visible in earlier studies which did not include maximizing behavior or realistic dynamics. My results show that ignoring these issues can result in welfare losses as rational orbit-users attempt to capture the rents created by launch taxes or dissipate the rents created by debris removal.[3]

I contribute to the literature on common-pool resource management and orbit use in three ways. First, I present the first economic analysis of orbit use management policy under open access. Prior models of orbit use with policy recommendations such as Bradley and Wein (2009), Weeden and Chow (2012), Macauley (2015), and Adilov, Alexander, and Cunningham (2015) do not simultaneously account for open access, forward-looking investment behavior and dynamics, and physical uncertainty in collision risk while developing policy prescriptions. Accounting for these features together reveals novel insights, such as the fact that regulating

[2]The legal issues of debris removal are non-trivial. International space law gives satellite operators ownership of their debris even after the satellite's lifetime, forcing potential salvage operations to negotiate with each individual satellite or fragment owner for rights to remove debris. For small fragments, attributing ownership and negotiating removal may be infeasible. A substantial legal and engineering literature has considered these issues and potential solutions, for example Carroll (2009); Merges and Reynolds (2010); Ansdell (2010). Many of these scholars have suggested amending existing legal frameworks to allow salvage and debris removal bounties, so as to incentivize negotiations between debris owners and would-be debris removers. Yet the march of technology continues despite legal uncertainty, and debris removal technologies are being developed and tested, for example Pearson, Carroll, and Levin (2010).

[3]To be clear, the need for orbit use management policy is not uniform across all orbits. Formal orbit allocation procedures exist in the geosynchronous (GEO) belt (Macauley, 1998; Jehn, Agapov, and Hernandez, 2005). But in low-Earth orbit (LEO), no such procedures exist. Orbital management must be done indirectly through spectrum management by national authorities (such as the Federal Communications Commission in the US), or directly through non-binding guidelines from international agencies (such as the Inter-Agency Debris Committee).

satellites in orbit is preferable to regulating satellite launches since satellite controls affect only the private marginal cost of launching while launch controls affect both the private marginal benefit and cost of launching.[4]

Second, I explicitly incorporate dynamic feedbacks and physical uncertainty in orbit use and study their economic effects. This allows my model to be augmented with high-fidelity engineering models and provide policymakers with quantitative policy design guidance. Prior models of orbit use such as Macauley (2015) and Adilov, Alexander, and Cunningham (2015) focus on qualitative properties of orbit use. While Macauley (2015) estimates tax and rebate values for a range of space traffic control policies, the values are derived in a two-period framework which obscures the effects of dynamic feedbacks, physical uncertainty, and open access launch behavior. Accounting for these effects is necessary to provide real-time quantitative guidance for optimal or second-best policies.

Third, I present the first economic analysis of the effects of active debris removal on orbit use, accounting for profit-maximization and open access. Prior models of active debris removal such as Liou and Johnson (2009b), Bradley and Wein (2009), Carroll (2009), and Ansdell (2010) do not account for economic behavior in studying the physical or legal issues in debris removal. Muller, Rozanova, and Urdanoz (2017) accounts for profit motives when deriving a lower bound on the value of debris removal, and Klima et al. (2016) accounts for physical dynamics and strategic behavior between debris removers when analyzing debris removal, but neither account for open access launch behavior or compare the resulting outcomes to first-best outcomes. My modeling framework accounts for profit-maximization and open access alongside physical dynamics and identifies previously-unknown issues in orbit use management, such as the relevance of how debris removal is financed to equilibrium outcomes and the inefficiency in cooperative removal plans created by open access launching.[5] My modeling framework can also provide quantitative guidance regarding optimal debris removal policy and the size of open access distortions to debris removal.

The remainder of the paper is organized as follows. In section 2 I describe institutional details of orbit use and present the basic definitions and modeling framework. In section 3 I analyze orbit management policies and the use of debris removal technologies, and present my key results: Proposition 5, that stock controls achieve greater expected social welfare than flow controls, and Proposition 6, that satellite owners must pay for debris removal if the technology is to reduce equilibrium collision risk. I show proofs of these and a few other economically

[4]There are similarities between the orbit use problem and other global commons or congestible resource problems, such as regulating atmospheric carbon dioxide or controlling road traffic congestion. Satellite controls are analogous to congestion pricing, while launch controls are analogous to road access tolls or gasoline taxes. Directly pricing congestion is more effective at controlling marginal road use decisions than pricing access or fuel.

[5]My analysis can be thought of as a "best-case" bound on the effects of debris removal: even if the strategic issues in debris removal identified by Klima et al. (2016) could be overcome, open access makes the cooperative removal plan continue to be inefficient.

important results in the main text (the rest are in the Appendix, section 5). Finally, I conclude in section 4 with discussion of the results and thoughts on the future of commercial orbit use.

2 Essentials of Orbit Use

In this section I discuss the history and current status of space traffic control policies. Readers interested in going directly to the modeling approach may skip to section 2.2. Readers interested in learning more of the institutional details of orbit use may go to the Appendix, section 8. The model of orbit use presented here is developed in a deterministic setting in Rao and Rondina (2018). However, the focus of that paper is very different from this one. Rao and Rondina (2018) focuses on developing economic intuition for satellite launching and orbit use, formally establishing the existence of the open access equilibrium, optimal launch plan, and negative externality, deriving the marginal external cost of orbit use, and analyzing the short-run and long-run dynamics of orbit use under open access. In this paper I include uncertainty over the collision rate and consider both policy choice and the use of active debris removal technologies.

An individual firm in this model must decide whether to launch a satellite or not. For tractability, I assume that each firm owns at most one satellite and that satellites are infinitely lived.[6] The firm's problem is contrasted with the decision facing the fleet planner, who owns all satellites and can launch as many as they would like each period. The planner is an "orbital monopolist" in this respect, and internalizes all of the marginal external cost of new satellites. Both the planner and firms take the price of satellite services as given.[7] Later in the paper, I expand the choice sets for satellite owners and the fleet planner to include the amount of debris removal they purchase. The details in sections 2.1 and 2.2 describe the environment these agents operate in and the challenges of space traffic control. Section 2.3 describes the agents themselves and the associated equilibrium or optimality conditions.

2.1 Defining "space traffic control"

One of the central challenges of space traffic control is how to define "space traffic control". Nicholas Johnson, a scientist at NASA, has proposed an aim of space traffic control: "...the goal of space traffic management is to minimize the potential for (radio frequency) or physical

[6]Relaxing the assumption of "one firm, one satellite" would mitigate the externality to some degree, as firms would be forced to internalize more of the collision risk and debris growth effects, at least to the extent that their own satellites were affected. However, it can introduce complicated best-response dynamics between firms, as debris and collision risk may weaken or remove competitor firms. Assuming that satellites are infinitely lived is qualitatively unimportant, easy to introduce, and studied in Rao and Rondina (2018).

[7]This is realistic for telecom services to most populated areas, as the terrestrial alternatives pin down the price. I do not study how an orbit user would manipulate collision risk and debris growth to alter the price of satellite services.

interference at any time" Johnson (2004). The radio frequency interference problem is relatively tractable and being handled by existing institutions (Jones et al., 2010). The physical interference problem, essentially collision avoidance, is more difficult from technical and legal perspectives. In GEO, space traffic control is "position control": since satellites in GEO have very low speeds relative to each other, traffic control is as simple as spacing satellites far enough apart that they are unlikely to collide or cause radio frequency interference. In the current regulatory regime, the International Telecommunications Union assigns frequency blocks and geostationary "slots" to national authorities. These authorities are then free to assign their frequencies and slots to entities within their jurisdiction as they see fit, and are also responsible for enforcing responsible spectrum use. In the United States, this is handled by the FCC.[8]

Space traffic control in LEO is harder than in GEO. Satellites in LEO are constantly in motion with respect to each other and have little or no control over their trajectories. Notions like "keep-out zones" are impractical since satellites may only occasionally or accidentally pass through them, and concepts like "rules of the road" raise the question of how a road is to be defined in LEO. Figure 1 shows the orbits of 56 cataloged satellites with mean altitudes of 700-710 kilometers, and makes the inaptness of road, sea, and air analogies clear. The growth in LEO use has motivated calls for broader notions of space traffic control which encompass non-GEO regimes. There are currently no international regulatory agencies which coordinate launches and satellite placements to manage debris growth and collision risk; the extent of management policies currently is a patchwork of national regulations and non-binding international guidelines. Table 1 shows the breakdown of currently-operational satellites by location of launch site to emphasize the international dimension of the problem. Figure 2 shows the growth in orbit use from active satellites and debris, as well as the increase in competition to provide commercial launch services.

[Figure 1 about here.]

[Table 1 about here.]

For this paper, I define space traffic control as policies or technologies intended to manage the probability of collisions between active satellites and other bodies. This definition encompasses satellite path as well as debris growth management. Any space traffic control policy, including command-and-control regulations, can be characterized as a price or quantity control, such as a tax or a quota. If the effect of a policy is to raise the cost or limit the availability of satellite launch, I label it as a "flow" control. If the effect is to raise the cost of operating a satellite or constrain the allowed number of satellites in orbit, I label it as a "stock" control.

[8]Readers interested in more detail about the history and institutions of space traffic control are referred to Johnson (2004); Jones et al. (2010). Technical proposals for mass removal are discussed in Klinkrad and Johnson (2009), Weeden (2010) discusses the legal challenges, and Tkatchova (2018) examines the potential for markets in debris removal.

The existing patchwork of policies includes both flow controls intended to manage launch capacity and prevent launches from interfering with air traffic, and stock controls intended to manage spectrum congestion. While most existing literature on space traffic control focuses on controlling the trajectories of objects in orbit, I focus on controlling the number of objects in orbit. Brief consideration will show that the former implies the latter. I treat debris removal separately because the technology is not yet commercially available, so analysis of a world without debris removal is more immediately relevant to policy design.

[Figure 2 about here.]

2.2 A simple model of orbital mechanics

In this section I describe the laws of motion for orbital stocks, the type of uncertainty most relevant to the economics of managing collision risk and debris growth (symmetric physical uncertainty), and the functional forms I use for simulations. Following analytical debris modeling studies such as Rossi et al. (1998) and Bradley and Wein (2009), I consider the evolution of orbital stocks in an arbitrary spherical shell around the Earth, referred to as the "shell of interest". More detailed physical models of Earth orbit use multiple shells. I ignore such features in this paper for tractability. I consider two types of fictitious agents: a social planner who launches and owns all satellites in orbit to motivate optimal satellite launch and debris removal plans, and a global regulator who manages all satellites launched or in orbit to motivate policy choice.

Let S_t denote the number of active satellites in orbit in period t, D_t the number of debris objects in orbit in t, X_t the number of launches in t, and ℓ_t the proportion of satellites which will be lost in collisions at the end of period t (the collision rate). δ is the proportion of debris objects which deorbit at the end of t (the decay rate), and $G(S_t, D_t, \ell_t)$ is the number of new debris fragments generated due to all collisions between satellites and debris. I assume that the collision rate is nonnegative and bounded below by 0 and above by 1.[9] No satellites can be destroyed when there are none in orbit ($S_t = 0 \implies \ell_t \equiv 0$). As $S_t \to \infty$ or $D_t \to \infty$, $\ell_t \to 1$ due to physical crowding (unless there are no satellites in orbit). I ignore the fact that active satellites may be deorbited when their useful lifetime is over, as it does not impact the economics of open access satellite launching. I consider the ability of different types of control policies to induce deorbits in Proposition 3. The effect of finite lifetimes is examined in Rao and Rondina (2018), and does not affect the principles of policy choice or debris removal use.

[9] Firms try to avoid collisions by maneuvering their satellites when possible; the collision rate in this model should be thought of as the rate of collisions which could not be avoided, with easily avoided collisions optimized away. Collisions which could have been avoided but were not due to human error are included in this. Implicitly I am assuming that firms operate their satellites as imperfect cost-minimizers.

The number of active satellites in orbit in t is the number of launches in the previous period plus the number of satellites which survived the previous period. The amount of debris in orbit is the amount from the previous period which did not decay, plus the number of new fragments created in collisions, plus the amount of debris in the shell created by new launches. Formally,

$$S_{t+1} = S_t(1-\ell_t) + X_t \tag{1}$$

$$D_{t+1} = D_t(1-\delta) + G(S_t, D_t, \ell_t) + mX_t. \tag{2}$$

I assume that the number of new fragments is nonnegative, increasing in each argument, and zero when there are no objects in orbit ($G(0,0,0) = 0$). δ is the rate of orbital decay for debris, and m is the amount of launch debris created by launching new satellites. To allow the possibility of Kessler Syndrome, I also assume that the growth in new fragments due to debris interactions alone ($G(0,D,0)$) will eventually be greater than the decay rate ($-\delta D$) if there are enough debris fragments (D large enough).

The most important source of uncertainty in orbit management is uncertainty over the proportion of satellites lost to collisions in a given period, ℓ_t.[10] However, the growth in debris objects is also uncertain, so why is uncertainty in $G()$ not treated similarly? The answer is uncertainty in orbit is of economic interest only insofar as it affects active satellites. Uncertainty in the position and interactions between objects in orbit is not economically relevant to orbit management. To see this, consider a counterfactual world where active satellites could not be affected by debris. In this case the uncertainty in debris growth would be irrelevant for orbit management, because the regulator's interest in orbit management is in controlling the value generated by active satellites. Thus, uncertainty in this model is represented by ℓ_t not because debris growth is known perfectly, but because uncertainty over orbital stocks only matters to the extent to which it affects active satellites. Definition 1 describes the revelation of ℓ_t.

Definition 1. *(Symmetric physical uncertainty) The collision rate in period t is revealed to all agents after any debris removal decisions are made but before any launch decisions are made for the period.*

"Symmetric physical uncertainty" means that launching firms and the regulator all know how many satellites will be lost in period t before acting, but not which satellites. On the other hand, satellite owners engaging in debris removal actions don't know how many satellites will

[10]Note that there are also sources of economic uncertainty in orbit use. The future trajectory of launch costs, demand for orbits from other industries, the decisions made by other operators to harden their satellites, and many other factors in orbit use are uncertain. I focus on physical uncertainty because of its first-order relevance to satellite survival and the usability of the orbital environment. Including sources of economic uncertainty would likely change the result in Proposition 1 to favor price or quantity instruments according to the relative slopes of marginal benefit and marginal cost curves (Weitzman, 1974).

be lost until after their removal action. The timing reflects three features of orbit use: (1) while conjunction alerts may be issued to affected operators up to a few days before an anticipated collision, longer-term forecasts of the collision environment are inherently probabilistic; (2) satellite owners who wish to remove debris will attempt to do so before the collisions are unavoidable; and (3) firms choosing whether or not to launch satellites can anticipate satellite owners' removal actions, in part because conjunction alerts are issued publicly to all satellite operators near the affected region so as to better coordinate avoidance maneuvers.

The symmetry between firms and the regulator is practically plausible: firms and regulatory agencies all have access to the same types of information about the position of orbital bodies[11], and can run similar calculations to predict the motion of orbital bodies from given position data. The US Department of Defense makes orbital object data fine enough to perform high-fidelity conjunction analysis on specific satellites available for nominal fees, while aggregate patterns can be modeled using data the Department of Defense makes publicly available. The European Space Agency makes similar data publicly available, albeit at lower fidelity for academic and hobbyist use. Most of these analyses are probabilistic in nature. Since satellites in the model are all identical, the identity of the satellites lost doesn't matter, and the probability any specific satellite is lost is the same as the aggregate rate.

I assume that ℓ_t has a conditional density $\phi(\ell_t|S_t, D_t)$. With physical uncertainty, only the density of ℓ_t is determined by S_t and D_t, so I explicitly include the draw of ℓ_t as an argument of $G()$. The expected value at the end of period t of a function $f(\ell_{t+1})$ is

$$E_t[f(\ell_{t+1})] = \int_0^1 f(\ell_{t+1})\phi(\ell_{t+1}|S_{t+1}, D_{t+1})d\ell_{t+1}. \tag{3}$$

I also assume that the distribution of the collision rate is "increasing" in the number of satellites and amount of debris, in the sense that an increase in either satellites or debris results in a new distribution which first-order stochastically dominates the old one. Assumption 1 states this precisely.

Assumption 1. *(The collision rate is increasing in satellites and debris) An increase in either satellites or debris results in a new distribution which first-order stochastically dominates the old one, that is,*

$$\int_0^k \ell\, \phi(\ell|S+\varepsilon_S, D+\varepsilon_D)d\ell \geq \int_0^k \ell\, \phi(\ell|S,D)d\ell \quad \forall \varepsilon_S, \varepsilon_D \geq 0,\ \forall k \in (0,1),$$

with strict inequality for some $\varepsilon_S > 0$ *or* $\varepsilon_D > 0$.

[11] State actors, particularly national security agencies, may have different information than other agents, breaking the symmetry. As long as the regulator(s) are equally ignorant of this information as firms, the symmetry holds. In some cases the regulator may actually have an informational advantage over firms. This suggests a question not pursued here: what are the limits to a regulator's ability to manage orbital congestion without revealing state secrets?

To reduce notational burden, I suppress the conditioning variables where it is clear from context, though I sometimes make them explicit in proofs. For brevity, I refer to $E_t[\ell_{t+1}]$ as the "collision risk".

2.2.1 Functional forms for the collision risk and number of new fragments

For simulations, I use functional forms for the collision risk and new fragment formation based on engineering model in Bradley and Wein (2009):

$$E[\ell|S,D] = \min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}, \tag{4}$$

$$G(S,D,\ell) = \begin{cases} \beta_{SS}\left(\frac{S}{S+D}\right)\ell S + \beta_{SD}\left(\frac{D}{S+D}\right)\ell S + \beta_{DD}\alpha_{DD}D^2 \text{ if } S+D>0 \\ 0 \text{ if } S+D=0 \end{cases} \tag{5}$$

which satisfy all the properties described above. α_{SS}, α_{SD}, and α_{DD} are positive constants which can be derived from an ideal gas model and descriptions of the shapes and sizes of the subscripted object types. They are often referred to as "intrinsic collision probabilities" in engineering studies. β_{SS}, β_{SD}, and β_{DD} are positive constants describing the mean "effective" (that is, adjusted for size and time spent in the shell of interest) number of fragments created in collisions between the subscripted object types. These can be calculated from descriptions of the material compositions of the objects colliding, their relative velocities, and masses. They are often referred to as "fragmentation parameters" in engineering studies. These forms are used to generate figures and simulations, but not for analytical results.

Economically, the expected collision risk can be thought of as a matching function which matches active satellites to debris and other active satellites. The form in equation 4 implies that matching between active satellites and debris or other active satellites exhibits "thick market effects": one more active satellite or unit of debris increases the ease with which all active satellites are matched with other orbital bodies. The economic intuition of the expected collision risk function is discussed in more detail in Rao and Rondina (2018).

2.2.2 Kessler Syndrome

Kessler Syndrome is a central concern in orbit use management. If open access can prevent Kessler Syndrome, regulating orbit use is not as important from an environmental perspective. Even though orbit use will be inefficient it will not cause irreversible environmental damage. On the other hand, if open access can cause Kessler Syndrome, orbit use management is more urgent.

In this section I formally define Kessler Syndrome and establish some properties of the debris threshold beyond which it occurs. Open access debris levels are increasing in the excess return on a satellite while the Kessler threshold is constant, implying that sustained increases in the return on a satellite can cause Kessler Syndrome under open access. Though the Kessler threshold is defined purely in terms of the system's physics, the occurrence of Kessler Syndrome depends critically on the economics of orbit use.

Assumption 2. *(Debris growth) The growth in new fragments due to debris is larger than the decay rate for all levels of the debris stock greater than some level $\bar{D} > 0$,*

$$\bar{D} : G_D(0, D, \ell) > \delta \; \forall D > \bar{D} \;\; \forall \ell.$$

Due to assumption 2 and $G(S, D, \ell)$ being increasing in all arguments, there is a unique threshold $D^\kappa \geq \bar{D}$ above which Kessler Syndrome occurs. Past this threshold, the number of new fragments created by collisions between debris exceeds the amount which decays in a single period. For regimes where this condition doesn't hold at any level of debris, Kessler Syndrome is impossible. Such regimes are likely to be at extremely low altitudes, possibly sub-orbital. For all of the simulations shown in this paper, Kessler Syndrome is possible.

Definition 2. *(Kessler Syndrome) The Kessler region is the set of debris levels for which cessation of launch activity and immediate deorbit of all active satellites cannot prevent continued debris growth, that is,*

$$D^\kappa : G(0, D^\kappa, \ell) > \delta D^\kappa \;\; \forall \ell.$$

Kessler Syndrome has occurred when the debris stock enters the Kessler region.

Without active debris removal technologies, Kessler Syndrome is an absorbing state. Once Kessler Syndrome occurs in the model, the debris stock grows without bound. In reality, the fragments would eventually pulverize each other into small fragments and either find stable orbits or decay back to the Earth, but this process could take centuries or millennia (Kessler and Cour-Palais (1978)).

2.3 The economics of open access and optimal orbit use without debris removal

With the environment and physical considerations described, I turn to the optimization problems facing orbit-using agents. I compare the equilibrium resulting from individual firms' launch decisions under open access to the outcome of the optimal launch plan followed by the fleet planner. The main point of this section is the comparison between equations 11 and 15, presented in figure 3.

An infinitely-lived firm which owns a satellite collects a return of π every period that the satellite survives and applies a discount factor of $\beta = \frac{1}{1+r}$ to future revenues. A fraction ℓ_t of the orbiting satellites are destroyed in collisions every period. The realization of ℓ_t is revealed to all agents just after satellites launched in $t-1$ have reached orbit, but before any launch decisions in t are made. Thus, ℓ_t is known before launch decisions in t, but ℓ_{t+1} is unknown. Expectations in period t are therefore taken over realizations of ℓ_{t+1}, as shown in equation 3. Since the satellites are identical, the probability that an individual satellite survives the period is $(1-\ell_t)$, and the probability it is destroyed is ℓ_t. If the satellite is destroyed, the firm will once again face the decision of launching or not. The value of a satellite in period t is

$$Q(S_t,D_t,\ell_t,X_t) = \pi + \beta[(1-\ell_t)E_t[Q(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})] + \ell_t E_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]] \tag{6}$$

A firm which does not own a satellite in period t faces the decision to pay a fixed cost F and launch a satellite which will reach orbit and start generating revenues in period $t+1$, or to wait and decide again whether or not to launch in period $t+1$. Once the firm decides to launch, it must wait one period before the satellite will reach orbit and begin producing returns. Assuming potential launchers are risk-neutral profit maximizers, the value of potential launcher i at period t is

$$\begin{aligned} V_i(S_t,D_t,\ell_t,X_t) = \max_{x_{it}\in\{0,1\}} &\{(1-x_{it})\beta E_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})] \\ &+ x_{it}[\beta E_t[Q(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})] - F]\} \qquad (7) \\ S_{t+1} &= S_t(1-\ell_t) + X_t \\ D_{t+1} &= D_t(1-\delta) + G(S_t,D_t,\ell_t) + mX_t \\ \ell_{t+1} &\sim \phi(\ell|S_{t+1},D_{t+1}) \end{aligned}$$

Under open access, firms launch until profits are zero:

$$X_t > 0: \; V_i(S_t,D_t,\ell_t,X_t) = 0 \tag{8}$$

$$\implies \beta E_t[Q(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})] = F. \tag{9}$$

The value of a satellite is then

$$Q(S_t,D_t,\ell_t,X_t) = \pi + (1-\ell_t)F, \tag{10}$$

and the equilibrium collision risk is

$$E_t[\ell_{t+1}] = r_s - r, \tag{11}$$

where $r_s = \frac{\pi}{F}$ is the one-period rate of return on a single satellite.[12]

[12]The open access equilibrium is Markov perfect: conditional on the state of the game, no launching firm can profit by deviating to "wait" and no waiting firm can profit by deviating to "launch".

With the open access equilibrium launch rate characterized, I move on to characterize the fleet planner's optimal launch rate. Recall that the planner owns all satellites in orbit, and can launch as many as they would like each period. The fleet planner maximizes the expected net present value of the entire fleet. Their problem is

$$W(S_t, D_t, \ell_t) = \max_{X_t \geq 0} \{\pi S_t - F X_t + \beta E_t[W(S_{t+1}, D_{t+1}, \ell_{t+1})]\} \tag{12}$$

$$\text{s.t.} \quad S_{t+1} = S_t(1 - \ell_t) + X_t \tag{13}$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t, \ell_t) + m X_t. \tag{14}$$

The planner launches so that the loss rate is equated to the rate of excess return net of the marginal external cost (ξ_{t+1}), that is,

$$E_t[\ell_{t+1}] = r_s - r - \frac{E_t[\xi(S_{t+1}, D_{t+1})]}{F}. \tag{15}$$

where $E_t[\xi(S_{t+1}, D_{t+1})]$ is the marginal external cost of a satellite launch. For the results in this paper, it suffices to assume that the marginal external cost is weakly positive for all S_{t+1} and D_{t+1} along the optimal path, and strictly positive for some values of S_{t+1} and D_{t+1}. Readers interested in the properties of the marginal external cost of satellite launches are referred to Rao and Rondina (2018), where $\xi(S_{t+1}, D_{t+1})$ is derived explicitly in a deterministic setting and shown to be positive under economically and physically intuitive conditions. Figure 3 illustrates the differences between open access and optimal policies in the deterministic setting.

[Table 3 about here.]

2.4 Debris removal technologies

Orbital debris is dangerous to active satellites in part because debris objects cannot be maneuvered and often do not transmit their location to ground stations. Active satellites, on the other hand, tend to do both, making collision avoidance maneuvering easier. Active satellites also tend to have some guidance and control systems which allow them to be deorbited remotely, if necessary. Debris objects tend not to have such systems, because they are fragments of a satellite, non-responsive to ground operator commands, or out of fuel and incapable of further maneuvers. Active debris removal technologies are those which can interact with debris objects and deorbit them. They are contrasted with passive removal, which involves measures like setting a satellite on a path which will result in its deorbit in a specified timeframe.

Active debris removal technologies are being developed, but have not yet been commercially deployed. Some of these technologies involve specialized removal satellites which use the Earth's magnetic field for propulsion and deploy nets, harpoons, or tethers (for example, Pearson, Carroll, and Levin (2010)) to either deorbit debris or recycle the materials for in-space manufacturing. Ground-based lasers are another candidate technology to deorbit debris.

I assume no new satellites are required to implement removal, which can be interpreted in two ways: that the removal technology is ground-based; or that the satellites required are already in orbit and can never be destroyed or lost. Including the requirement that new satellites be used for removal complicates the model in interesting and relevant ways that are beyond my scope here. I also assume that only satellite owners can purchase debris removal.

With the ability to remove debris from orbit, satellite owners can remove clearly-dangerous pieces of debris before they impact their satellites. The remaining collisions will be caused by errors in debris risk assessments, satellite trajectory forecasts, and collisions which were deemed too costly to avoid. To reflect this in the model, I adjust the timing of when ℓ_t is revealed when debris removal technologies are present. Satellite owners purchase R_t total units of removal before ℓ_t is revealed, with the aim of changing the distribution of ℓ_t until the marginal private benefit of removal equals the marginal private cost. After removal has been purchased, ℓ_t is drawn from a distribution conditioned on S_t and $D_t - R_t$ (instead of just S_t and D_t) and revealed to all satellite owners and prospective launchers. The launchers then decide whether or not to launch.[13]

With debris removal before collisions, the laws of motion and distribution of the collision rate become

$$S_{t+1} = S_t(1-\ell_t) + X_t \tag{16}$$

$$D_{t+1} = (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t \tag{17}$$

$$\ell_t \sim \phi(\ell_t | S_t, D_t - R_t). \tag{18}$$

Expectations before removal in t are indicated by $\tilde{E}_t[\cdot]$ and treat ℓ_t as a random variable, while expectations after removal in t are indicated by $E_t[\cdot]$ and treat ℓ_t as known. The expected collision risk before removal is effected is

$$\tilde{E}_t[\ell_t] = \int_0^1 \ell_t \phi(\ell_t | S_t, D_t - R_t) d\ell_t. \tag{19}$$

Potential launchers in t have the same expectations as before: they are aware of ℓ_t, and treat ℓ_{t+1} as uncertain. Formally,

$$E_t[\ell_{t+1}] = \int_0^1 \ell_{t+1} \phi(\ell_{t+1} | S_{t+1}, D_{t+1} - R_{t+1}) d\ell_{t+1} = \tilde{E}_{t+1}[\ell_{t+1}]. \tag{20}$$

Though $E_t[\ell_{t+1}] = \tilde{E}_{t+1}[\ell_{t+1}]$, I use separate notation so that the subscript on the expectation

[13] In reality, the timing of satellite launches and debris removals will not be this clearly separated. However, potential launchers will be able to anticipate satellite owners' debris removal demands, and where possible structure their launches to take advantage of these efforts.

operator indicates the period in which the agent forms the expectation, and the tilde above the expectation operator indicates whether the expectation is formed before or after is drawn and revealed. $E_t[\ell_{t+1}]$ is an expectation formed in t after ℓ_t is drawn and revealed, $\tilde{E}_t[\ell_t]$ is an expectation formed in t before ℓ_t is drawn and revealed.

3 Space Traffic Control

3.1 Policy without active debris removal

Space traffic control policies restrict either the number of satellites launched to or the number of satellites in an orbit in a given window of time. As described earlier, I refer to policies restricting the number of launches in a given period as *flow controls*, and policies which restrict the number of satellites in orbit in a given period as *stock controls*. Stock controls entail an explicit or implicit payment made every period that the satellite is in orbit. The payment gives the satellite owner the right to keep their satellite in orbit that period. Flow controls entail a payment made once when the satellite is launched. The payment gives the satellite launcher the right to launch in that period. Table 2 gives some examples of each type of control policy. Both types of controls are currently in place around the world - the FAA's launch permit system is a flow control for launches from the United States, while the ITU's minimum spacing requirements for satellites in GEO are a stock control for GEO use.[14] Existing controls tend to be implemented as quantities, as in the two examples given, but could also be implemented as prices, for example, a launch or satellite tax.

[Table 2 about here.]

Quantity restrictions imply price restrictions and vice versa. In many settings, either mode can generate equivalent social welfare. Weitzman (1974) establishes that the equivalence can break down in the presence of regulatory uncertainty over the firm's marginal cost of production. Whether a price or quantity instrument should be preferred in such settings depends on the relative slopes of the marginal benefit and marginal cost curves. This is not the case for orbits, where the main source of uncertainty comes from the motion of physical objects which are in principle observable by all actors. Unlike the regulatory problems considered in Weitzman (1974) and Newell and Pizer (2003), the firm has no additional information about the motion of orbital bodies for the regulator to harness through instrument design.

The distinction between stock and flow controls is relevant to a broad class of economic management problems. To encourage renewable energy generation, a regulator may weigh

[14] These requirements tend to focus on launch capacity and spectrum interference, rather than the risk of collisions and debris growth. The framework developed here applies to orbit use management policies regardless of their intent.

investment (stock) vs production (flow) tax credits (Aldy, Gerarden, and Sweeney, 2018). To manage public infrastructure a regulator may weigh investment in damage abatement (flow) vs quality restoration (stock) (Keohane, Van Roy, and Zeckhauser, 2007).[15]

In the absence of informational or administrative constraints on the regulator, the preferred instrument is that which most directly targets the externality-generating activity (Sandmo, 1978). In the renewable energy case, production tax credits can encourage renewable energy generation more effectively than investment tax credits.[16] In orbit, stock controls dominate flow controls because the collision risk externality is driven by the number of objects in orbit rather than the number of objects launched in a period. Because stock controls directly target the incentive to own a satellite while flow controls target the incentive to launch a satellite, they present satellite owners and launchers with different incentives. These differing incentives drive a wedge between their abilities to manage orbital congestion.

Stock and flow controls can often be made equivalent in the sense that one can be capitalized or annuitized to the same present value cost as the other. However, they have different effects on the incentive to launch or own a satellite. Imposing a fee at launch increases the cost of entering the orbital commons, penalizing entrants while increasing the rents accruing to incumbents in orbit. Imposing a recurring fee while the satellite is in orbit reduces the rents of satellite ownership without restricting entry, treating entrants and incumbents equally. These differing incentives can lead to welfare differences between stock and flow modes of orbit control. To show how stock and flow controls affect the decision to launch a satellite, consider two cases with price-based controls. In the first, a stock control is levied on satellite owners. In the second, a flow control is levied on satellite launchers. I assume the regulator can commit to future policies, so that $t+1$ values are known to firms with certainty.[17]

[15] Keohane, Van Roy, and Zeckhauser (2007) consider the use of stock and flow controls to manage the quality of a resource, but their use of "stock control" is slightly different due to the setting considered. In their setting, "stock controls" refer to policies which restore the stock of a deteriorating resource. Here, the term refers to limiting the stock of a commodity which deteriorates the resource. Keohane, Van Roy, and Zeckhauser (2007)'s use of "flow controls" is closer to the use of the term here: they consider abating the flow of pollutants into the environment, and I consider controlling the flow of satellites into orbit.

[16] Provided capacity is not a binding constraint, production effort is costly, and the production function is not characterized by decreasing returns to scale, as described in Aldy, Gerarden, and Sweeney (2018) and Parish and McLaren (1982).

[17] Stock and flow controls both require forward guidance, since announced or anticipated $t+1$ values affect the launch rate in t. But whereas flow controls require forward guidance regarding the entire time path of control values, stock controls only require forward guidance about the next-period control value. Even without commitment, the regulator faces no incentive to deviate from a previously-announced stock control rule. This may not be the case for flow controls. Because anticipated changes in the flow control rule can cause launching firms to "bunch" and attempt to launch either just before a price increase or just after a price decrease, the regulator has an incentive to make flow control policy changes a surprise. Such surprises would change private expectations of the control policy path. In environmental economics, Newell, Pizer, and Zhang (2005) consider the tradeoffs between commitment and discretion in stabilizing quantity-policy prices. This tradeoff is analyzed in more depth in the monetary policy literature; Svensson (2003) provides a comprehensive discussion.

The decision to launch under a stock control: Let the price that a satellite owner pays in t be p_t^s. Firms deciding whether to launch or not in period t will account for their anticipated regulatory burden as they drive the profits of launching a satellite down to zero. The marginal benefit from owning a satellite in $t+1$ must therefore equal not only the opportunity cost of the launch, but also the direct regulatory cost of the stock control. Formally,

$$X_t : \beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = F \tag{21}$$

$$E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = \pi - p_{t+1}^s + (1 - E_t[\ell_{t+1}])F \tag{22}$$

$$\implies \pi = rF + E_t[\ell_{t+1}]F + p_{t+1}^s. \tag{23}$$

The decision to launch under a flow control: Let the price that a satellite launcher pays in t be p_t^f. Firms deciding whether to launch or not in period t will account for the regulatory burden of launching. Since they know that future launchers will face a similar regulatory burden, they will consider how the future flow control price will affect the open access value of a satellite. The marginal benefit of owning a satellite in $t+1$ must therefore equal the opportunity cost of the launch, which includes the flow control price they pay and the forgone interest. However, the marginal benefit of owning a satellite in $t+1$ now includes not only the direct revenues the satellite generates but also the additional expected value from the flow control levied on $t+1$ launchers. Formally,

$$X_t : \beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = F + p_t^f \tag{24}$$

$$E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = \pi + (1 - E_t[\ell_{t+1}])F + (1 - E_t[\ell_{t+1}])p_{t+1}^f \tag{25}$$

$$\implies \pi + (1 - E_t[\ell_{t+1}])p_{t+1}^f = rF + E_t[\ell_{t+1}]F + (1+r)p_t^f. \tag{26}$$

Figure 4 illustrates equations 23 and 26. Although imposing either type of control can reduce the equilibrium number of launches, flow controls raise the private marginal benefit of launching along with the private marginal cost. Stock controls, on the other hand, affect only the marginal cost of launching. This is the core intuition for why stock controls are preferable to flow controls for managing orbital congestion.

[Figure 4 about here.]

Leakage issues and legal hurdles: Both types of controls face leakage issues. Flow controls implemented by regional launch providers may suffer "launch leakage", while stock controls implemented by regional regulatory agencies may suffer "mission control leakage". Similar leakage issues have been studied extensively in the environmental and public economics literatures, for example Fowlie (2009); Fischer and Fox (2012); Böhringer, Rosendahl, and Storrøsten (2017). Though these issues are relevant to effective policy implementation, analyzing them is beyond the scope of this paper. The legal hurdles to implementing stock controls may also be higher than those for flow controls, since they require a legal framework in which the right to exclude agents from an orbit can be held and enforced. Such a framework would have to be globally agreed-upon and potentially self-enforcing. I do not consider the prospects of such an agreement in this paper, although similar issues have been studied extensively in economics generally and environmental economics specifically, for example Telser (1980); Barrett (2005, 2013).

3.1.1 Using stock and flow space traffic control policies

In this section, I formally describe some properties of stock and flow controls and how they should be used to manage space traffic. The first property is price-quantity equivalence: under symmetric physical uncertainty, a stock or flow control can be implemented as a price or quantity and achieve equivalent expected social welfare. This allows me to consider price or quantity implementations interchangeably. I then show how stock and flow controls should be used to limit launches, and consider the implications of these details for optimal control values. I follow this by showing how the launch rate responds to the initiation of a stock or flow control, and how a regulator could use those controls to induce firms to deorbit already-orbiting satellites and stop launching new ones. These properties are used in the following section to establish that regulating orbit use through stock controls achieves higher expected social welfare than using flow controls.

Proposition 1. *(Price-quantity equivalence) Under symmetric physical uncertainty, price and quantity implementations of stock controls are equivalent, as are price and quantity implementations of flow controls.*

Proof. I show the result for stock controls first, and then for flow controls.

Stock controls: I refer to price-based stock controls as satellite taxes, and quantity-based stock controls as satellite permit quotas. Let the launch rate under a satellite tax be $\tilde{X}_t$, and the permit price under a permit quota be $\tilde{p}_{t+1}$.

Under a satellite tax, the number of satellites launched will be

$$\tilde{X}_t : \pi = rF + E_t[\ell_{t+1}]F + p_{t+1} \tag{27}$$

Under a binding satellite permit quota, firms will purchase permits and launch satellites until the price of a permit is

$$\tilde{p}_{t+1} : \pi = rF + E_t[\ell_{t+1}]F + \tilde{p}_{t+1} \tag{28}$$

For a given state vector (S_t, D_t, ℓ_t) and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 27 determines a unique value of $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 28.

Flow controls: I refer to price-based flow controls as launch taxes, and quantity-based flow controls as launch permit quotas. Let the launch rate in t under a launch tax be $\tilde{X}_t$, and the permit price in $t+1$ under a permit quota be $\tilde{p}_{t+1}$.

Under a launch tax, the number of satellites launched will be

$$\tilde{X}_t : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])p_{t+1} \tag{29}$$

Under a binding launch permit quota, firms will purchase permits and launch satellites until the price of a permit is

$$\tilde{p}_{t+1} : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])\tilde{p}_{t+1} \tag{30}$$

For a given state vector (S_t, D_t, ℓ_t) and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 29 determines a unique value of $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 30. □

With access to commitment, a regulator using a flow control sets either the future number of permits or their price (X_{t+1} or p_{t+1}) in order to influence the launch rate today (X_t). Raising p_{t+1} in t raises the marginal benefit of launching a satellite today, but lowers it tomorrow. The use of flow controls requires the regulator to trade off the future launch disincentive of raising p_{t+1} against the current launch incentive it creates. The regulator's true instrument with a flow control is not the price of the control itself, but the *change* in price between periods. Rather than a price mapping to a quantity, here it is a (real) change in price which maps to a quantity and vice versa. The regulator can set any initial flow control price so long as they commit to a path of control prices based on equation 26. A similar penalty-rebate structure appears in the mining flow control studied in Briggs (2011), where incentivizing mine owners to mine less in t requires a lower Pigouvian tax in period $t+1$.

Note that stock control prices must be positive to reduce launches in any given period, while flow control prices need not be positive to do the same. Along positive price paths the flow control is an entry restriction while along negative price paths it is an entry subsidy. Current restrictions deter current entry, but future restrictions deter future entry and boost the rents accruing to incumbents, incentivizing current entry. Current subsidies encourage current entry, but future subsidies encourage future entry and reduce the rents accruing to incumbents, incentivizing firms to delay entry. In either case, the regulator is able to use the change in flow control prices to rearrange satellite launches over time.[18]

[18] Technically, the flow control structure creates a problem if the loss rate drawn in t is large enough that the expected loss rate in $t+1$ is one. If this happens, there is no future launch control price which can satisfy equation 26. Lemma 8 in section 9.6 of the Appendix shows this formally. If the regulator wishes to control the launch rate in periods where the expected future loss rate is one, they must break their earlier commitment and adjust p_t until the launch rate is where they want it to be. Since I assume the regulator cannot break their commitment, in this case there is simply no time-consistent flow control

The need to commit to a flow control path makes terminal conditions economically relevant to their use. If the regulator plans to use the flow control for only a limited duration, after which the orbits will be under open access again, the flow control price path will decrease over time until it is zero in the period where open access is restored.[19] A flow control which attempts to ensure optimality with no planned phase-out will be forced to follow an exploding price path, positive or negative, as the regulator attempts to balance present and future incentives and disincentives without causing launchers to "bunch" suboptimally in any period while the control is active. This property of the price path is formally established in section 9.6 of the Appendix. The credibility of such a price path is doubtful, but beyond my scope here.

Limiting launches with stock and flow controls: While stock controls are straightforward - raise the price to reduce launches - flow controls are subtler. To limit launches in t, the flow control price in $t+1$ should be lowered instead of raised. The intuition for this can be seen in Figure 4 and in equation 26, where the price of a flow control in $t+1$ enters the launch decision in t as a marginal benefit rather than a marginal cost. This has implications for the design of optimal controls: an optimal stock control equates the $t+1$ control price with the expected marginal external cost in $t+1$, while an optimal flow control makes the expected real difference in t and $t+1$ control prices equal to the negative of the expected $t+1$ marginal external cost.

Lemma 1. *(Launch response to stock and flow controls) The open access launch rate is*

- *decreasing in the future price of a stock control;*
- *decreasing in the current price and increasing in the future price of a flow control.*

Proof. See Appendix section 5. □

Corollary 1. *The shift in marginal cost of owning a satellite due to an increase in the flow control price is greater than the prior shift in marginal benefit due to the entry restriction.*

Proof.

$$r > 0 \implies \left|\frac{\partial X_t}{\partial_t p}\right| > \left|\frac{\partial X_t}{\partial p_{t+1}}\right| \tag{31}$$

$$\implies 1 + r > 1 - E[\ell_{t+1}], \tag{32}$$

which can affect the launch rate. This holds for both price and quantity implementations. That said, if potential launchers expect their satellite to be destroyed after one period, they will only launch in the unrealistic edge case where one period of returns from a satellite exceeds the cost of launching. It is more likely that potential launchers would rather not launch if their satellites are expected to survive only one period.

[19] Why might a regulator want to do this? Open access launching tends to overshoot the open access steady state, potentially ending up in the Kessler region. A regulator who wished to prevent this without committing to optimality may therefore impose a flow control until the risk of overshooting is sufficiently reduced.

which is true because $\ell_{t+1} \in [0,1]$ by definition. □

Committing in t to raising the flow control price in $t+1$ raises the marginal benefit of owning a satellite before $t+1$, when the new price comes into effect and raises the marginal cost of launching a satellite. This increases the number of launches in t and reduces the number in $t+1$. On the other hand, committing in t to lowering the flow control price in $t+1$ reduces the marginal benefit of owning a satellite in $t+1$, when the new price comes into effect and lowers the marginal cause of launching a satellite. This reduces the number of launches in t and increases the number in $t+1$. This "launch bunching" is absent in stock controls.

Optimal control policies: Making a stock control optimal is simple: set the price equal to the expected marginal external cost of another satellite. Letting p_t^s be the value of the stock control in period t,

$$p_{t+1}^s = E_t[\xi(S_{t+1}, D_{t+1})] \tag{33}$$

will make launchers behave as the planner would command.

Making a flow control optimal is more complicated. The value of the control in the previous period must be taken into account to balance intertemporal launch incentives. The expected survival rate must be accounted for as well, as it determines the expected rent due to entry restriction the firm will realize. Formally, letting p_t^f be the value of the flow control in period t,

$$p_{t+1}^f = \frac{(1+r)p_t^f - E_t[\xi(S_{t+1}, D_{t+1})]}{1 - E_t[\ell_{t+1}]} \tag{34}$$

is required. Perhaps counterintuitively, the expected marginal external cost of another satellite must be *subtracted* from the future value of the stock control. This is because the future flow control price represents a benefit to current launchers, rather than a cost, as seen in Figure 4. Figure 7 shows examples of optimal stock and flow control policies.

Initiating control: Along an interior equilibrium path, stock and flow controls can both be optimal. This raises the questions of whether the equivalence holds when a control is first put into place, and whether boundaries (periods when a control is implemented from an open access status quo, or when a control is used to shut down all launches) present any challenges to either control type. Proposition 2 shows that the equivalence does not hold at initiation boundaries. Figure 5 illustrates Proposition 2.

Proposition 2. *(Smoothness at boundaries) Stock controls can be initiated without letting the launch rate exceed the open access launch rate. Flow controls cannot be initiated without forcing the launch rate to exceed the open access launch rate.*

Proof. See Appendix section 5. □

[Figure 5 about here.]

As described in Lemma 1 and Figure 4, positive flow control prices first shift the marginal benefit of owning a satellite before the control is implemented upwards, then shift the marginal cost of owning a satellite after the control is implemented upwards. As described in Corollary 1, the increase in marginal cost is necessarily greater than the increase in marginal benefit, so if the flow control price is kept stationary after the increase, there will be fewer launches per period than before. However, more than the equivalent open access number of firms will launch just before the flow control price is raised to capture its rents.

Price and quantity stock controls are equally easy to use to halt all launching. As a price, the control value is simply raised until no firm wants to launch. As a quantity, the control value is simply frozen at whatever number of satellites in orbit us desired. Using flow controls for this purpose is trickier, with quantities being more intuitive than prices. A quantity flow control can be used to halt all launching by setting the number of allowed launches to zero. Though expectations of a launch shutdown may induce firms to launch earlier, the mechanics are described in Lemma 1 and bunching can be mostly avoided with careful attention to the entire path of allowed launch quantities (bunching before the period when control is implemented is unavoidable). Using a price flow control is not as simple as raising the price once, however, since (a) in the period before the price increase firms will want to launch to capture the rents from restricted entry the following period, and (b) if the difference in the flow control price between periods is constant launching may resume. To avoid bunching and maintain shutdown, the regulator must instead *lower* the flow control price in the period when launch shutdown is desired, and then commit to an ever-decreasing sequence of prices which will eventually go to negative infinity (the reasoning is described in Lemma 7). The credibility of such price paths is questionable.

Inducing satellite owners to deorbit: Satellite owners often have the option to deorbit their satellite if it becomes too expensive to operate[20]. In this section only, I include the deorbit option for satellite owners to consider whether stock and flow control policies can induce deorbits. The firm's net payoff from deorbiting their satellite is $V^d \lesseqgtr 0$. V^d includes any liquidation revenues (for example, from selling mission control equipment) or costs (for example, costs of damage to people or property during the deorbit). Firms decide whether or not to deorbit after ℓ_t is revealed. A firm which decides to deorbit doesn't claim the revenues

[20]In the early 2000s, lack of profitability nearly induced the operators of the Iridium constellation to deorbit their satellites. Iridium SSC ultimately went bankrupt, but was able to find a consortium of buyers who kept the constellation in orbit. Modern cubesats are often launched without sufficient guidance and control capabilities to initiate deorbit. Their trajectories are typically planned so that they will naturally deorbit within a few years of their launch.

from being in orbit that period. Formally,

$$Q(X_t, S_t, D_t, \ell_t) = \max\{\pi + (1-\ell_t)F, V^d\} \tag{35}$$

Satellites that are in the process of being deorbited may still collide with each other or be struck by debris. Denoting the number of satellites deorbited as Z_t, the laws of motion with deorbit are

$$S_{t+1} = (S_t - Z_t)(1-\ell_t) + X_t \tag{36}$$

$$D_{t+1} = D_t(1-\delta) + G(S_t, D_t, Z_t, \ell_t) + mX_t \tag{37}$$

$$\ell_t \sim \phi(\ell_t | S_t, D_t). \tag{38}$$

The satellites which are deorbited but still destroyed in collisions ($Z_t\ell_t$) are included in D_{t+1}. Firms choose to deorbit if the payoff from deorbit exceeds the payoff from remaining in orbit:

$$\text{Deorbit if } V^d > \pi + (1-\ell_t)F \tag{39}$$

$$\ell_t > 1 + \frac{\pi - V^d}{F}. \tag{40}$$

$V^d < \pi$ is a no-arbitrage condition: it ensures that firms can't pump money out of an orbit by repeatedly launching satellites and deorbiting them as soon as they reach orbit.[21] The no-arbitrage condition implies that flow controls can't force firms to deorbit with positive flow control prices, described in Proposition 3.

Proposition 3. *(Controlling the rate of deorbit) Stock controls with positive prices can make satellite owners deorbit their satellites and induce net deorbits (more deorbits than launches). Flow controls with positive prices cannot make satellite owners deorbit their satellites or induce net deorbits.*

Proof. See Appendix section 5. □

[21] Since $\ell_t \in [0,1]$, a firm will never deorbit its satellite if it isn't required. Since firms here own only one satellite each, it would be economically strange for them to throw away the potential for future profits by deorbiting. When firms own multiple satellites they may decide to deorbit one satellite to preserve others. If the satellites depreciate or technology improves, they may decide to deorbit and replace a satellite. In both cases the no-arbitrage condition $V^d < \pi$ would still apply. Suppose a constellation owner with depreciating satellites will deorbit one of their satellites, j, at time $\bar{t}_j$ in response to the depreciation and collision risks between j and the other satellites in their constellation. Accounting for its marginal external cost on other satellites outside the constellation, j should be deorbited at time $t_j^* \lesseqgtr \bar{t}_j$. Whether the socially optimal deorbit time is sooner or later than the privately optimal deorbit time will depend on the relation between the effects of satellites and launch debris on collision risk and debris growth. If the marginal effect on collision risk of a satellite exceeds the marginal effect of new launch debris, $t_j^* < \bar{t}_j$. A satellite-specific stock control with price p_{jt} would be able to ensure that $t_j^* = \bar{t}_j$ for all j in each constellation. In the case of Iridium, π fell below V^d and (some of) its operators believed it would stay there.

Intuitively, making it costlier for firms to launch new satellites cannot make already-orbiting satellites less valuable. This is why flow controls are unable to induce deorbits, at least with positive prices. Flow controls with negative prices may or may not be able to induce deorbits, depending on parameter values and the number of new entrants induced.

3.1.2 Risks and policy choice

In this section, I consider how the choice of stock or flow control mode will affect the equilibrium collision risk and the probability of Kessler Syndrome. I establish that stock controls generate weakly higher expected fleet values than flow controls over arbitrary horizons. Due to the smoothness properties described in Proposition 2, both collision and Kessler risks are increased when a flow control is initiated but not when a stock control is initiated. Fundamentally, the stock of objects in orbit is the source of orbit use externalities, not the flow of objects into orbit. Because of this, it is intuitive that stock controls are better tools to manage space traffic than flow controls. An analogy to road traffic control is instructive here. Road traffic congestion is, in theory, better regulated by congestion pricing levied than by congestion-based road access tolls since congestion pricing affects marginal road use decisions throughout the road while access tolls affect marginal road use decisions only at points of entry.

Proposition 4. *(New stock controls reduce risk and debris, a new flow controls increase them) The equilibrium expected collision risk, the expected future debris stock, and the probability of Kessler Syndrome will*

- *decrease when a generic stock control is introduced;*
- *increase when a generic flow control is introduced.*

Proof. The effect of introducing a control on the equilibrium collision risk: Suppose a control is scheduled to be introduced at date t. The equilibrium collision rate under open access in period $t-2$ is

$$\hat{X}_{t-2} : E_{t-2}[\ell_{t-1}] = r_s - r. \tag{41}$$

In general, the equilibrium expected future collision rate is an increasing function of the current launch rate. Proposition 2 establishes that $X_{t-1} < \hat{X}_{t-1}$ (the equivalent uncontrolled open access launch rate in $t-1$) if the control scheduled to be introduced in t is a stock control. Similarly, Proposition 2 establishes that $X_{t-1} > \hat{X}_{t-1}$ if the control scheduled to be introduced in t is a flow control. Thus, introducing a generic stock control must reduce the equilibrium expected future collision rate, while introducing a generic flow control must increase it.

The effect of introducing a control on the expected future debris stock: The debris stock in t is an increasing function of both the collision rate and launch rate in $t-1$:

$$D_t = (1-\delta)D_{t-1} + G(S_{t-1}, D_{t-1}, \ell_{t-1}) + mX_{t-1}. \tag{42}$$

Since the launch rate decreases when a stock control is introduced, D_t is mechanically reduced due to the reduction in launch debris (mX_{t-1}). Similarly, when a flow control is introduced, the increased launch rate increases D_t through launch debris. Even without launch debris, the same conclusion holds in the following period because the expected collision risk is an increasing function of the launch rate. Formally, suppose $m = 0$:

$$E_{t-1}[D_{t+1}] = (1-\delta)D_t + E_{t-1}[G(S_t, D_t, \ell_t)] \tag{43}$$

Because the expected number of new fragments is linear in probabilities,

$$E_{t-1}[D_{t+1}] = (1-\delta)D_t + G(S_t, D_t, E_{t-1}[\ell_t]) \tag{44}$$

$$\implies \frac{\partial E_{t-1}[D_{t+1}]}{\partial p_t} = \frac{\partial G(S_t, D_t, E_{t-1}[\ell_t])}{\partial p_t} \tag{45}$$

$$= \frac{\partial G(S_t, D_t, E_{t-1}[\ell_t])}{\partial E_{t-1}[\ell_t]} \frac{\partial E_{t-1}[\ell_t]}{\partial X_{t-1}}. \tag{46}$$

Both terms on the right-hand side of the final line are positive: the expected number of new fragments formed in collisions is increasing in the expected number of collisions, and the expected number of collisions is increasing in the number of satellites launched the previous period.

The effect of introducing a control on the probability of Kessler Syndrome: The probability of Kessler Syndrome occurring in t, given information in $t-1$, is

$$Pr_t(D_t(\ell_{t-1}) > D^{\kappa}). \tag{47}$$

We have already established that for any ℓ_{t-1}, D_t will decrease if a stock control is implemented in t, and increase if a flow control is implemented in t. D^{κ} is a function of the physical parameters of the orbit, and is unaffected by economic controls. Consequently, introducing a stock control must reduce the probability of Kessler Syndrome, while introducing a flow control must increase the probability of Kessler Syndrome. □

The result in Proposition 4 is one of the main reasons why a regulator should prefer stock controls to flow controls. If the imposition of a control raises the equilibrium collision risk, it is possible that it may also cause Kessler Syndrome. The economic intuition for this effect is simple. Flow controls generate rents for firms who already own satellites. Imposing a flow control therefore creates an incentive for marginal launchers to become satellite owners before the flow control is imposed. One way around this would be to levy a flow control with no prior notice. I do not consider this possibility, as it would force firms to form expectations over the regulator's possible actions. These expectations may be difficult for the regulator to elicit truthfully, as firms might anticipate the regulator's desire to act in unexpected ways to reduce firms' profits. Such expectations would make it difficult to implement regulatory policy effectively. Stock controls sidestep this issue by focusing on satellite owners. Forward-looking launchers internalize their expected costs due to the control, and can be appropriately disincentivized against launching without distorting the incentives of current satellite owners.

The relative advantage of stocks vs. flows: The question of ultimate interest to a regulator is likely one of policy choice: "which type of instrument is better, and why?" The results so far - particularly Proposition 4 - suggest that stock controls should be preferred to flow controls along generic paths. Proposition 5 compiles the results so far to answer the policy choice question along optimal paths. Since stock controls can be initiated without losing control of the launch rate and induce deorbits when necessary, they can achieve first-best outcomes in every state of the world. Flow controls cannot. Even if interior launch rates are optimal forever and no deorbits are ever required, flow controls will achieve less social welfare than stock controls when they are put into place.

Proposition 5. *(The relative advantage of stocks vs. flows) The expected social welfare under an optimal stock control strictly exceeds the expected social welfare under an optimal flow control for an arbitrary horizon where a control must be initiated, used to stop all launches, or used to force net deorbits.*

Proof. The fleet welfare from both controls can be equal along interior equilibrium paths. However, when the flow control is initiated, Proposition 2 shows that it will cause the launch rate to exceed the uncontrolled open access launch rate whereas a stock control would not. Proposition 4 shows that the launch bunching from initiating a flow control will also cause the risk of Kessler Syndrome to increase. In those periods, stock controls will achieve strictly greater expected social welfare than flow controls.

Proposition 3 shows that flow controls may not be able to induce net deorbits (never with positive prices and only possibly with negative prices), while stock controls can always do so. Therefore, for arbitrary paths with positive prices where the regulator must either initiate control, shut down orbital access, or induce net deorbits, stock controls achieve strictly greater expected social welfare than flow controls. □

Proposition 5 is fairly straightforward, and may even understate the advantages of stock controls over flow controls. From a computational perspective, optimal flow controls are much harder to implement than optimal stock controls because they require attention to the entire control time path. Lemma 7 in the Appendix shows that price-based flow controls must have an exploding price path to balance the launch incentives and disincentives described in Lemma 1 and Figure 4. One solution to this may be to use a quantity flow control, such as a launch permit quota system. However, the regulator must still commit to a time path of quantity policies when the flow control is implemented, cannot prevent launch bunching before the policy goes into effect, and cannot induce deorbits. Stock controls face none of these issues. A one-period-forward forecast of the marginal external cost is sufficient, which would have been required anyway under a flow control. The regulator faces no commitment issues and can precisely control the number of satellites in orbit at any given time.

3.1.3 Optimal space traffic control policies

Before finishing my discussion of stock and flow controls, I illustrate optimal control policy functions by simulation. For clarity and computational tractability, I use deterministic simulations where $E_t[\ell_{t+1}] = L(S_{t+1}, D_{t+1})$. Figure 7 shows an example of optimal stock and flow control policies as a satellite tax and a launch tax. Figure 6 shows the underlying satellite stocks, debris stocks, and launch rates used to compute Figure 7.

The point of figures 6 and 7 is to show the qualitative properties of optimal stock and flow controls. While both types of tax vary with the marginal external cost of launching a satellite ($E_t[\xi(S_{t+1}, D_{t+1})]$), only the satellite tax varies positively with the marginal external cost. This is a convenient feature for applying stock controls: the behavior of an optimal stock control is more intuitive than that of an optimal flow control. The reasoning behind this behavior is described in in Lemma 1 and Figure 4.

[Figure 6 about here.]

[Figure 7 about here.]

3.2 Active debris removal and open access

I now turn to the effects of active debris removal technologies on orbit use. My main result, Proposition 6, shows that while active debris removal can mechanically reduce the debris stock no matter how it is financed, it can only reduce the equilibrium risk of satellite-destroying collisions to the extent that satellite owners pay for debris removal. I show this in two steps. First, I show that exogenously provided removal which is free to satellite owners will reduce the debris stock but increase the satellite stock. The increase in the satellite stock will exactly offset the decrease in risk from debris removal, leaving the equilibrium collision risk unchanged. Then, I consider a case where exogenous debris removal involves a mandatory fee paid by satellite owners. I show that as the fee goes to zero, the collision risk returns to the original open access level.

Lastly, I show how endogenously chosen debris removal purchased by cooperative satellite owners reduces the debris stock, collision risk, and risk of Kessler Syndrome, while also allowing more firms to launch satellites. These results depend on some auxiliary properties of cooperative debris removal and open access launching with debris removal, shown in the Appendix, section 6.3. Though the jointly-optimal launch and removal plan is analytically complicated, I simulate the fleet planner's launch and removal plans and compare them to the launch and removal plans under open access and cooperative removal. The simulations show that cooperative decentralized removal plans are similar to the planner's removal plans, though the launch plans differ more substantially. While the cooperative decentralized removal plan matches the regulator's in many regions of the state space, the firms do not begin debris removal as quickly as the planner would. Open access launchers respond to satellite and debris in orbit very differently than the planner: while the planner effectively ignores debris in choosing launch rates, open access firms are disincentivized by debris and not sufficiently responsive to satellites, at least until the open access launchers expect cooperative satellite owners to begin debris removal. Economically, open access launchers do not work with satellite owners to coordinate their launch activity due to their inability to secure property rights over orbits. Their launch behavior prevents cooperative satellite owners from internalizing the full value of debris removal, thus inducing the satellite owners to begin debris removal later than the planner would.

3.2.1 An economic model of active debris removal

With debris removal technology available, satellite owners purchase R_{it} units of removal from a competitive debris removal sector. The price of a unit of removal is c_t. The amount of debris removed is nonnegative and cannot exceed the total amount of debris in orbit. Since firms are identical, each satellite owner will choose the same level of removal, making the total amount of debris removed $R_t = S_t R_{it} \leq D_t$. The maximum amount that an individual satellite owner could choose to remove is D_t/S_t. To consider the best-case outcomes of debris removal, I focus on cooperative removal plans between satellite owners. I establish an economically intuitive necessary and sufficient condition for cooperation to be locally self-enforcing in section 6.1 of the Appendix. The condition is stated below in Assumption 3.

Assumption 3. *(Making cooperation locally self-enforcing) For any non-zero cooperative removal plan, the change in the equilibrium collision risk before debris removal is greater than the ratio of the removal price to the launch cost, that is,*

$$\left.\frac{\partial \tilde{E}[\ell_t|S_t,\bar{D}-R_t]}{\partial \bar{D}}\right|_{\bar{D}=D_t} > \frac{c_t}{F} \quad \forall R_t \in [0,\bar{D}].$$

While the validity of Assumption 3 will need to be evaluated empirically for specific technologies and orbital regimes, I assume it always holds in the analysis below.[22]

The value of a satellite after debris has been removed and ℓ_t has been drawn is

$$Q_i(S_t,D_t,\ell_t,X_t) = \pi + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1},D_{t+1}) + \ell_t E_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]]. \tag{48}$$

[22]For example, cooperation may be enforced by a grim trigger mechanism under which any deviation by any firm results in no debris removal at all.

The value of a satellite owner who purchases debris removal before the loss is

$$\begin{aligned}
\tilde{Q}_i(S_t,D_t) &= \max_{0\leq R_{it}\leq D_t/S_t}\{-c_tR_{it}+\tilde{E}_t[Q_i(S_t,D_t,\ell_t,X_t)]\} \\
\text{s.t. } \ell_t &\sim \phi(\ell_t|S_t,D_t-R_t) \\
Q_i(S_t,D_t,\ell_t,X_t) &= \pi+\beta[(1-\ell_t)\tilde{Q}_i(S_{t+1},D_{t+1})+\ell_tE_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]] \\
S_{t+1} &= S_t(1-\ell_t)+X_t \\
D_{t+1} &= (D_t-R_t)(1-\delta)+G(S_t,D_t-R_t,\ell_t)+mX_t.
\end{aligned} \tag{49}$$

The value of a launcher is

$$V_i(S_t,D_t,\ell_t,X_t) = \max_{x_{it}\in\{0,1\}}\{(1-x_{it})\beta E_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]+x_{it}[\beta\tilde{Q}_i(S_{t+1},D_{t+1})-F]\} \tag{50}$$

$$\begin{aligned}
\text{s.t. } \tilde{Q}_i(S_t,D_t) &= \max_{0\leq R_{it}\leq D_t/S_t}\{-c_tR_{it}+\tilde{E}_t[Q_i(S_t,D_t,\ell_t,X_t)]\} \\
\ell_t &\sim \phi(\ell_t|S_t,D_t-R_t) \\
Q_i(S_t,D_t,\ell_t,X_t) &= \pi+\beta[(1-\ell_t)\tilde{Q}_i(S_{t+1},D_{t+1})+\ell_tE_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]] \\
S_{t+1} &= S_t(1-\ell_t)+X_t \\
D_{t+1} &= (D_t-R_t)(1-\delta)+G(S_t,D_t-R_t,\ell_t)+mX_t.
\end{aligned}$$

Under a generic launch plan, the decision to remove debris is dynamic: removal today will impact the amount of debris tomorrow through the number of satellite destructions and the number of debris-debris collisions. Under open access, the value of a satellite tomorrow will always be driven down to the current value of the launch cost, so the future benefits of removal will never accrue to today's satellite owners. This makes the removal decision under open access static: the only benefit of debris removal internalized by satellite owners today is the way that it changes the probability that their satellite is destroyed. Even though the cost of removal is linear, nonlinearity in the coupling between the debris stock and the collision rate can yield an interior solution to the removal decision. This simplifies analysis of cooperative removal plans given open access launch plans.

Open access launching: Under open access, firms will launch satellites until the value of launching is zero:

$$\forall t,X_t:\ V_i(S_t,D_t,\ell_t,X_t)=0 \tag{51}$$

$$\implies \beta\tilde{Q}(S_{t+1},D_{t+1})=F \tag{52}$$

$$\implies Q_i(S_t,D_t,\ell_t,X_t)=\pi+(1-\ell_t)F. \tag{53}$$

Taking R_t as fixed, and assuming that launchers plan to choose R_{it+1} optimally when they are satellite owners, the flow condition determining the launch rate is

$$\pi = rF + \tilde{E}_{t+1}[\ell_{t+1}]F + R_{it+1}c_{t+1}. \tag{54}$$

This can be rewritten to yield the equilibrium collision risk,

$$\tilde{E}_{t+1}[\ell_{t+1}] = r_s - r - \frac{c_{t+1}}{F}R_{it+1}. \tag{55}$$

Equation 55 states that the equilibrium collision risk will be equal to the excess return on a satellite ($r_s - r$) minus the rate of total removal costs the launchers will face when they become satellite owners ($\frac{c_{t+1}}{F}R_{it+1}$). If there were no removal technology, $R_t = 0\ \forall t$, and the equilibrium collision rate would be equal to the excess return on a satellite. In any period t, the decisions to launch and to remove debris are undertaken by different firms. Potential launchers make the launch decisions, while current satellite owners make the removal decisions. Once they become satellite owners, launchers will face the removal decision. While open access makes satellite owners myopic, satellite launchers remain forward-looking.

Cooperative private debris removal: Profit maximizing cooperative satellite owners will demand debris removal until their marginal benefit from removal equals its marginal cost. Under open access to orbit, the first-order condition for an interior solution to the maximization problem in system of equations 49 is

$$R_{it} : c_t = \frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t} S_t F, \tag{56}$$

with the second-order condition

$$R_{it} : -\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2 F < 0. \tag{57}$$

Intuitively, open access removes any potential future benefit or cost from debris removal. Satellite owners will not get to reap any benefits from increasing Q_{it+1} because today's potential launchers will enter and capture them. Equation 56 therefore states that under open access launching, satellite owners will purchase debris removal until the price of a unit of removal (c_t) is equal to the static private marginal benefit ($\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t} S_t F$). That benefit has three pieces: the value of their satellite next period, F; the number of owners who will make the same removal decision, S_t; and the change in the probability that their satellite is destroyed at the end of the period, $\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}$.

Under open access firms launch until zero profits, while satellite owners remove debris until marginal benefits equal marginal costs. But current launchers are future satellite owners. If they could not coordinate as launchers, how can they do so as satellite owners? The answer is property rights. International space law gives satellite launchers ownership of any objects they put into space, even after their useful life is over. As a result, satellite owners must either purchase or exercise rights to specific pieces of debris in order to remove them. This allows satellite owners to coordinate debris removal. I assume they do so in a cooperative and efficient manner to focus on the best-case scenario for active debris removal. I ignore both the complications of decentralized bargaining between many parties and the difficulties of attributing ownership to specific small pieces of debris. Transaction and information costs associated with debris removal are relevant to policy design and implementation, but beyond my scope here.

3.2.2 Exogenous debris removal for free and for a mandatory fee

To develop intuition for how debris removal can reduce the equilibrium collision risk, consider a setting where $\bar{R}$ units of debris are removed from orbit every period by a regulator. Such policies are advocated for by some in the space debris literature, for example Bradley and Wein (2009) and Akers (2012). If the removal is costly to satellite owners, it is because the regulator forces them to pay a fixed fee of $\bar{c}$ per unit removed. Denote the equilibrium collision risk with exogenous removal for a mandatory fee as $E_t^R[\ell_{t+1}]$, and the equilibrium collision risk with no removal as $E_t[\ell_{t+1}]$. The open access equilibrium condition for forward-looking launchers is then

$$\pi = rF + \tilde{E}^R_{t+1}[\ell_{t+1}]F + \bar{R}\bar{c}, \tag{58}$$

while in the absence of removal, firms would launch until

$$\pi = rF + \tilde{E}_{t+1}[\ell_{t+1}]F. \tag{59}$$

Inspecting the two equations above reveals that, as the mandatory removal fee approaches zero, the equilibrium collision risk with removal approaches the equilibrium collision risk without removal. Proposition 6 shows this formally.

Proposition 6. *(Satellite owners must pay for collision risk reduction) Any debris removal technology will reduce the equilibrium collision risk if and only if:*

1. *some amount of debris is removed, and*
2. *satellite owners pay for the removal.*

Proof. Let the equilibrium collision risk with debris removal be $\tilde{E}^R_{t+1}[\ell_{t+1}]$. The amount of debris removed per satellite owner is $\bar{R}$, and the per-unit cost to satellite owners is $\bar{c}$. From equation 58,

$$\tilde{E}^R_{t+1}[\ell_{t+1}] = r_s - r - \frac{\bar{R}\bar{c}}{F}. \tag{60}$$

From equation 11, the equilibrium collision risk without debris removal is[23]

$$\tilde{E}_{t+1}[\ell_{t+1}] = r_s - r. \tag{61}$$

If no debris is removed, $\bar{R} = 0$. If some debris is removed but satellite owners pay nothing for it, $\bar{c} = 0$. In either case, $\tilde{E}^R_{t+1}[\ell_{t+1}] = \tilde{E}_{t+1}[\ell_{t+1}]$. If and only if some debris is removed ($\bar{R} > 0$) and satellite owners pay something for it ($\bar{c} > 0$), $\tilde{E}^R_{t+1}[\ell_{t+1}] < \tilde{E}_{t+1}[\ell_{t+1}]$.

More generally, for any positive amount of debris removal, the equilibrium collision risk reduction is increasing in the amount that satellite owners pay for debris removal:

$$\forall \bar{R} > 0, \quad \tilde{E}_{t+1}[\ell_{t+1}] - \tilde{E}^R_{t+1}[\ell_{t+1}] = \frac{\bar{R}\bar{c}}{F}, \tag{62}$$

$$\frac{\partial(\tilde{E}_{t+1}[\ell_{t+1}] - \tilde{E}^R_{t+1}[\ell_{t+1}])}{\partial \bar{c}} = \frac{\bar{R}}{F} > 0. \tag{63}$$

As firms pay less and less for debris removal, the equilibrium collision risk with debris removal smoothly approaches the equilibrium collision risk without debris removal:

$$\lim_{\bar{c} \to 0} \tilde{E}^R_{t+1}[\ell_{t+1}] = \lim_{\bar{c} \to 0} (\pi - rF - \bar{R}\bar{c}) = \pi - rF = \tilde{E}_{t+1}[\ell_{t+1}]. \tag{64}$$

□

Since the debris stock will be lower due to removal, the launch rate will be higher with free exogenous removal than it would under open access with no removal. Economically, free removal clears up space for new launchers to enter the orbit. This case highlights the mechanism through which active debris removal can reduce the equilibrium collision risk: not by mechanically reducing the amount of debris in orbit, but by reducing the excess return of a satellite. This mechanism also acts in the case with endogenous debris removal, as shown in Proposition 7. Despite this mechanism, the launch rate may be larger with debris removal than without. While the reduction of excess return on a satellite will lower the launch rate, the reduction in debris will increase the launch rate.

This example also highlights the main reason why active debris removal can reduce collision risk: not because it removes debris, but because it approximates a stock control. This suggests that controls on debris removal could be more effective than flow controls on satellites at reducing collision risk. Figure 8 illustrates the differences between exogenous debris removal for free and for a mandatory fee. When satellite owners choose how much debris to remove, however, this type of approach must account for current satellite owners' and launchers' responses to the price of removal. These responses are discussed in Propositions 5 and 13.

[Figure 8 about here.]

[23]While the expectation in equation 11 looks slightly different from the expectation in equation 61, $\tilde{E}_{t+1}[\ell_{t+1}]$ is the same integral as $E_t[\ell_{t+1}]$. The difference in notation is described in equation 20.

3.2.3 Endogenous debris removal financed by satellite owners

Figure 9 illustrates the effects of introducing active debris removal paid for by cooperative satellite owners. Unlike when removal is provided exogenously, endogenous removal can induce more firms to launch satellites before the technology becomes available. Despite the cost of cooperating with others and paying for removal, lower expected collision costs due to debris removal and lower individual contributions due to additional firms paying for removal makes it optimal for potential launchers to enter the orbit.

[Figure 9 about here.]

The introduction of debris removal technologies affects equilibrium orbital stocks as well as open access launch incentives. I explore the properties of cooperative private debris removal demands and open access launching further in the Appendix, section 6.3. Two of these - the uniqueness of the cooperatively-optimal post-removal debris stock and the potential for a "dynamic virtuous cycle" of debris removal - are driven by the incentives of satellite owners given open access. The third result describes intuitive physical and economic conditions under which the demand for satellite ownership by satellite launchers will be decreasing in the launch cost. Violations of these conditions may be plausible depending on the values of physical parameters.

3.2.4 Cooperative removal and open access risks

Ultimately, policymakers considering active debris removal technologies will want to know how debris removal will affect collision and Kessler Syndrome risks. In this section, I examine how active debris removal which is costly to satellite owners will change the equilibrium collision risk, the equilibrium future debris stock, and the equilibrium future probability of Kessler Syndrome. The results of this section, Propositions 7 and 8, establish that the use of active debris removal can reduce both equilibrium collision risk and the risk of Kessler Syndrome. While the risk of Kessler Syndrome can be mechanically reduced by removing debris no matter how removal is financed, reducing the equilibrium collision risk requires satellite owners to finance debris removal.

Proposition 7 extends Proposition 6 by considering the time path of collision risk when debris removal is introduced and when debris removal is ongoing. The intuition is similar to that of Proposition 6. Proposition 8 relies on some auxiliary properties of debris removal, shown in section 6.3 of the Appendix. The key intuition is that the cooperatively-optimal level of post-removal debris is a constant. As a result, even if there is an increase in the number of satellites due to debris removal, the risk of Kessler Syndrome will be reduced because firms will continuously purchase removal to keep the debris stock at its new, lower level.

Costly debris removal services can reduce the equilibrium collision risk: The removal of debris, all else equal, should reduce the collision rate. Whether it reduces the equilibrium collision risk depends on how potential satellite launchers respond to this reduction in risk. If debris removal spurs enough new entry, debris removal may result in higher collision rates. The logic seems plausible: if debris removal should reduce the collision rate, then more firms would be able to take advantage of the cleaner orbit and should therefore launch.

This logic would be correct but for an important detail: firms which launch satellites at t will become firms which own satellites at $t+1$. As they decide whether to launch or not, forward-looking firms account for their expected debris removal expenses as satellite owners. If the firm anticipates wanting to purchase debris removal services once its satellite is on orbit, and these services are costly, the introduction of the technology must reduce the excess return from launching the satellite. Since open access equates the expected collision risk with the excess return, the reduction in excess return also reduces the expected collision risk. However, if debris removal is introduced as a free service which potential launchers anticipate not paying for, the equilibrium collision risk will remain at the earlier open access level.

Proposition 7. *(ADR can reduce collision risk) The introduction of costly debris removal services in period t will reduce the equilibrium collision risk in t if and only if*

1. *it is costly to remove debris in t, and*
2. *it is privately optimal for cooperative satellite owners to remove some amount of debris in t.*

Ongoing active debris removal will reduce the equilibrium collision risk if and only if individual cooperative debris removal expenditures increase from period t to t + 1. Formally,

$$E_{t-1}[\ell_t] - E_t[\ell_{t+1}] > 0 \iff c_{t+1}R_{it+1} > c_t R_{it}.$$

Proof. The proof of this result is similar to the proof of Proposition 6, so I omit it from the main text. See Appendix section 5. □

At first, the collision risk will decrease because of the expenditure that current launchers anticipate making once they are satellite owners. The fact that debris removal directly removes orbital debris is incidental to this risk reduction. As in the exogenous case, open access drives new launchers to take advantage of the newly-cleared space by launching satellites until the risk is the same as it was before removal because available. Subsidies for debris removal to satellite owners may increase the equilibrium amount of debris removed, but would not affect the equilibrium collision risk. Open access will still dissipate rents from orbit use; subsidized debris removal would only tilt the combination of new satellites and debris which equilibriates the system toward new satellites. Ongoing debris removal can only keep the collision rate below the no-ADR open access collision rate if and only if it is costly to potential launchers. If potential launchers anticipate that the cost will reduce, or that it will not be optimal to purchase removal as satellite owners, the collision rate will return to the no-ADR open access level.

Cooperative costly debris removal will reduce the equilibrium probability of Kessler syndrome: Since preventing Kessler Syndrome is one of the key motivations for developing active debris removal technologies, it is natural to wonder if debris removal will achieve this goal. Since Kessler Syndrome is caused by the amount of debris exceeding a threshold, debris removal in t will reduce the probability of Kessler Syndrome in $t+1$ if it is certain to reduce the $t+1$ debris stock. More precisely, the change in probability of Kessler Syndrome in $t+1$ is equal to the probability that the change in the $t+1$ debris stock due to removal in t is positive, plus the product of the change in expected collision risk due to removal and the original probability of Kessler Syndrome. Since the change in the $t+1$ debris stock due to removal is negative with probability one, debris removal will reduce future probability of Kessler Syndrome.

Proposition 8. *(Debris removal will reduce the future probability of Kessler Syndrome) Debris removal in t will reduce the probability of Kessler Syndrome in $t+1$.*

Proof. Kessler Syndrome will occur in $t+1$ when

$$D_{t+1} - D^{\kappa} > 0, \tag{65}$$

where D^{κ} is the Kessler threshold. Suppose that Kessler Syndrome has not already occurred ($D_t - D^{\kappa} < 0$). Under the probability density for satellite-destroying collisions in t ($\phi(\ell_t|S_t, D_t - R_t)$), the probability in t of Kessler Syndrome in $t+1$ is

$$Pr_t(D_{t+1} - D^{\kappa} > 0|S_t, D_t - R_t) = \int_0^1 \mathbb{1}(D_{t+1}(\ell_t) - D^{\kappa} > 0)\phi(\ell_t|S_t, D_t - R_t)d\ell_t. \tag{66}$$

The change in this probability due to an increase in R_t is

$$\frac{dPr_t(D_{t+1}(\ell_t) - D^{\kappa} > 0|S_t, D_t - R_t)}{dR_t} = \frac{\partial}{\partial R_t}\int_0^1 \mathbb{1}(D_{t+1}(\ell_t) - D^{\kappa} > 0)\phi(\ell_t|S_t, D_t - R_t)d\ell_t \tag{67}$$

$$\begin{aligned} &= \int_0^1 \mathbb{1}(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t} > 0)\phi(\ell_t|S_t, D_t - R_t)d\ell_t \\ &\quad + \int_0^1 \mathbb{1}(D_{t+1}(\ell_t) - D^{\kappa} > 0)\frac{\partial \phi(\ell_t|S_t, D_t - R_t)}{\partial R_t}d\ell_t \end{aligned} \tag{68}$$

$$\begin{aligned} &= Pr_t\left(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t} > 0|S_t, D_t - R_t\right) \\ &\quad - \left.\frac{\partial Pr_t(D_{t+1}(\ell_t) - D^{\kappa} > 0|S_t, \tilde{D} - R_t)}{\partial \tilde{D}}\right|_{\tilde{D}=D_t}. \end{aligned} \tag{69}$$

The first term in equation 69 is zero because debris removal reduces the future debris stock for any draw of the collision rate, and the form of the second term in equation 69 follows from Lemma 2. To see that the first term is zero, define the open access launch rate as an implicit function $X_t = X(S_t, D_t - R_t, \ell_t)$ defined by equation 54. Then, differentiate D_{t+1} with respect to R_t:

$$D_{t+1}(\ell_t) = (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t$$
$$\implies \frac{\partial D_{t+1}(\ell_t)}{\partial R_t} = -\left[1 - \delta + \frac{\partial G(S_t, D_t - R_t, \ell_t)}{\partial D_t} + m\frac{\partial X_t}{\partial D_t}\right].$$

From Proposition 11 and applying the Implicit Function Theorem to equation 54,

$$\frac{\partial X_t}{\partial D_t} = -\frac{\frac{\partial R_{it+1}}{\partial D_{t+1}}(1-\delta+\frac{\partial G(S_t,D_t-R_t,\ell_t)}{\partial D_t})(1-\frac{\partial R_t}{\partial D_t})}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right) + \frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}}$$
$$= 0 \text{ whenever } R_{it} > 0 \because \frac{\partial R_t}{\partial D_t} = 1 \text{ from Proposition 11}.$$

Therefore,

$$\frac{\partial D_{t+1}(\ell_t)}{\partial R_t} = -\left[1 - \delta + \frac{\partial G(S_t, D_t - R_t, \ell_t)}{\partial D_t}\right] < 0 \;\; \forall \ell_t \in [0,1].$$

Since the statement holds for all possible realizations of ℓ_t,

$$Pr_t\left(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t} > 0 | S_t, D_t - R_t\right) = 0.$$

The change in the probability of Kessler Syndrome due to a change in debris removal is then

$$\frac{dPr_t(D_{t+1}(\ell_t) - D^\kappa > 0 | S_t, D_t - R_t)}{dR_t} = -\left.\frac{\partial Pr_t(D_{t+1}(\ell_t) - D^\kappa > 0 | S_t, \tilde{D} - R_t)}{\partial \tilde{D}}\right|_{\tilde{D}=D_t}. \quad (70)$$

The right hand side of equation 70 is the negative of the change in the probability of Kessler Syndrome from the shift in the distribution of collision rates which a marginal amount of debris would cause. It is not precisely the same as the effect of another unit of debris, since the debris argument of $D_{t+1}(\ell_t)$ is held constant while the debris argument of $\phi(\ell_t|S_t, D_t - R_t)$ is increased slightly. By Assumption 1, increasing the amount of debris in orbit will shift the conditional density of the collision rate toward 1. The fact that $\mathbb{1}(D_{t+1}(\ell_t) - D^\kappa > 0)$ is at least weakly increasing in ℓ_t, combined with Lemma 4, the change in probability must be at least weakly positive. So, debris removal must reduce the probability of Kessler Syndrome:

$$\left.\frac{\partial Pr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, \tilde{D} - R_t)}{\partial \tilde{D}}\right|_{\tilde{D}=D_t} \geq 0 \tag{71}$$

$$\implies \frac{dPr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, D_t - R_t)}{dR_t} \leq 0. \tag{72}$$

□

Overall, debris removal technologies financed by satellite owners will make orbits safer, reducing both equilibrium collision risk and the risk of Kessler Syndrome. These gains in safety come from satellite owners financing the debris removal. Subsidized or publicly provided debris removal cannot reduce the equilibrium collision risk. Though it may reduce the equilibrium debris stock, subsidized or publicly provided debris removal may not reduce the equilibrium risk of Kessler Syndrome unless the agency providing debris removal commits to preventing the debris stock from exceeding a fixed level. Cooperative removal was shown to achieve this in Proposition 11. The value of debris removal technologies depends critically on the economic institutions under which the technologies are used.

3.2.5 Optimal removal and launch plans

Finally, I consider the jointly-optimal debris removal and satellite launch plans to compare with the cooperative removal and open access launch plans. The main point of this section is to show that even with cooperative debris removal financed by satellite owners, open access launching is not socially optimal. Proposition 10 establishes that a constrained planner cannot improve on the cooperative removal plan given open access, so this section is an exercise in determining how large a distortion open access launching creates. The unconstrained fleet planner coordinates removals and launches, taking advantage of the fact that they will be able to remove any unwanted debris before the collision rate is drawn. Their problem at the start of period t is

$$\begin{aligned}
\tilde{W}(S_t, D_t) &= \max_{R_t \in [0, D_t]} \{-c_t R_t + \tilde{E}_t[W(S_t, D_t - R_t, \ell_t)]\} \qquad (73)\\
\text{s.t. } W(S_t, D_t - R_t, \ell_t) &= \max_{X_t \geq 0} \{\pi S_t - F X_t + \beta \tilde{W}(S_{t+1}, D_{t+1})\}\\
\ell_t &\sim \phi(\ell_t | S_t, D_t - R_t)\\
S_{t+1} &= S_t(1 - \ell_t) + X_t\\
D_{t+1} &= (D_t - R_t)(1 - \delta) + G(S_t, D_t - R_t, \ell_t) + m X_t.
\end{aligned}$$

The planner faces the same timing of information as firms do: at the beginning of a period, before ℓ_t has been revealed, they choose how much debris they will remove. Based on their removal decision, the draw of ℓ_t is revealed. Then they decide how much they will launch. The program in system of equations 73 shows this decision-making process at the beginning of a period. Their jointly-optimal removal and launch plans must equate the social marginal costs and benefits of removing debris before ℓ_t is known and of launching satellites once ℓ_t is known. Formally,

$$R_t^* : c_t = -\left\{ \tilde{E}_t \left[\frac{\partial W(S_t, D_t - R_t, \ell_t)}{\partial D_t} \right] + \frac{\partial \tilde{E}_t [W(S_t, D_t - R_t, \ell_t | S_t, D - R_t)]}{\partial D} \bigg|_{D=D_t} \right\} \tag{74}$$

$$X_t^* : \frac{F}{\beta} = \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} + m \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}} \tag{75}$$

An optimal removal plan exists if the sum of the objects inside the curly brackets on the right side of equation 74 is positive. I assume that the marginal post-removal value of debris is negative ($\frac{\partial W(S_t, D_t - R_t, \ell_t)}{\partial D_t} < 0$) along optimal paths, making both terms on the right hand side individually negative. The negativity of the second term follows from Lemma 4. This is sufficient to make the right side of equation 74 positive.

An optimal launch plan exists if the sum of the objects on the right side of equation 75 is positive. I assume that along optimal paths, the marginal pre-removal value from another satellite is positive and the marginal pre-removal value from another piece of launch debris is negative (a loss). I also assume that the pre-removal gain from another satellite is larger than the pre-removal loss from another piece of launch debris ($\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} \geq 0$, $\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}} < 0$, $\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} > -m \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}}$).

To compare the qualitative properties of optimal removal and launch plans with cooperative removal and open access launch plans, I simulate cases of the cooperative removal and open access launch plans (system of equations 50) and the optimal removal and launch plan (system of equations 73). Figure 10 shows these plans and the associated value functions.

[Figure 10 about here.]

Comparing Figure 10 with Figure 3 shows that the open access launch plan with debris removal is similar to the plan without debris removal. When cooperative satellite owners will not remove debris, open access launchers reduce launching as the expected collision rate increases, stopping all launching when the expected collision rate exceeds the excess return on a satellite. When open access launchers anticipate cooperative satellite owners removing debris, they begin launching again at a rate independent of the amount of debris. This jump is shown in the time paths in Figure 9. The jump occurs because debris removal by incumbent satellite owners allows new firms to enter the orbit. Since the planner keeps the debris stock at a constant level as soon as the fleet value justifies it, they ignore debris while launching. The net effect is that with debris removal technologies available, open access firms may launch too many or too few satellites relative to the planner. Under open access, firms launch too many satellites when the expected collision rate is low because they don't internalize the marginal external cost of their satellites, and too few as the expected collision rate increases because they don't account for debris removal. When open access launchers anticipate debris removal, they once again launch too many satellites because incumbent satellite owners can't exclude the launchers from taking advantage of the newly-cleaned orbits. Eventually, the profits of owning a satellite net of the cost of debris removal is no longer sufficient to justify launching.

The cooperative debris removal plan and the planner's removal plan are both corner solutions once debris removal starts.[24] The planner, however, begins debris removal with fewer satellites than the cooperative firms. Intuitively, the planner starts removing debris once the fleet is valuable enough to justify removal, while cooperative satellite owners start removing debris once there are enough owners sharing the removal costs to justify removal.

The discontinuity in the open access launch plan, its dependence on the debris stock, and the later-than-optimal start to cooperative debris removal all reduce the value of the open access-cooperative fleet relative to the planner's. The value loss from open access launching and cooperative debris removal follows the launch plan deviation and is intensified along the removal plan deviation. The gap is maximized just before open access launchers, anticipating removal, begin to launch again. At that point, the planner would have stopped launching and have begun removing debris while cooperative satellite owners would still be waiting for more contributors.

4 Conclusion

In this paper I showed how principles of economics should guide our stewardship of orbital resources. I established the equivalence of price and quantity instruments for orbital management and showed why space traffic controls should target satellite ownership rather than satellite launches. I considered the impacts of using active debris removal technology, and showed why, to reduce equilibrium collision risk, satellite owners must pay for debris removal. While satellite-focused policies can achieve first-best orbit use, attempts to control orbital debris growth and collision risk through launch fees or debris removal subsidies under open access may be ineffective or backfire.

Along the way I derived practically-useful results about orbit use management under physical uncertainty with and without active debris removal. These include how to use stock and flow space traffic controls, the fact that debris removal can induce more launches no matter how it is financed, and the possibility that the open access launch rate may be increasing in the launch cost. I also examined the effects of indirect orbit control through spectrum regulation

[24] See Appendix section 6.2 for more details on nonconvexities and corner solutions in debris removal. The functional form in equation 4 implies that the marginal benefit of debris removal is constant with respect to debris, so it is optimal to either remove all debris or none.

or mandatory satellite insurance. These policies approximate stock controls and are potential avenues by which regulators can induce first-best orbit use.

Knowing these details will help regulators manage orbit use effectively. However, questions will only grow as humans develop a larger presence in space. Commercial satellite operators are increasingly using many small satellites arrayed in constellations to deliver services. How should satellite constellations be regulated? International agreements will be necessary to regulate orbit use and minimize leakages, but different nations have different interests. What kinds of international orbit use management agreements are incentive-compatible? Militaries are among the most prominent orbit users, and have objectives which may conflict with commercial operators or each other. How can strategic orbit use by militaries be efficiently managed without compromising national and international security objectives? These are all important directions for future research.

Satellites are important to the modern world. We depend on satellite telecommunications to reach remote parts of the planet, enabling telemedicine and timely rescue efforts. We depend on GPS for navigation, and will rely on it more as automated transportation infrastructure develops. We depend on satellite imagery to determine the extent of natural disasters and optimize our responses to them, which climate change will only make more necessary. Economists are well-positioned to apply and develop the lessons of this paper, preventing a tragedy of the orbital commons and enabling a new wave of economic growth.

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5 Appendix A: Proofs and technical details

5.1 Proofs not shown in the main text

Lemma 1 (Launch response to stock and flow controls): *The open access launch rate is*

- *decreasing in the future price of a stock control;*
- *decreasing in the current price and increasing in the future price of a flow control.*

Proof. Stock controls: From equation 23, we can write

$$\mathscr{I} = \pi - rF - E_t[\ell_{t+1}]F - p_{t+1} = 0. \tag{76}$$

Applying the Implicit Function Theorem, we get that

$$\begin{aligned}
\frac{\partial X_t}{\partial p_{t+1}} &= -\frac{\partial \mathscr{I}/\partial p_{t+1}}{\partial \mathscr{I}/\partial X_t} &(77)\\
&= -\frac{-1}{-\frac{\partial E[\ell_{t+1}]}{\partial X_t}} &(78)\\
&= -\frac{1}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}} &(79)\\
&= -\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} < 0. &(80)
\end{aligned}$$

Flow controls: From equation 26, we can write

$$\mathscr{G} = \pi - rF - E_t[\ell_{t+1}]F - (1+r)p_t + E_t[(1-\ell_{t+1})p_{t+1}] = 0. \tag{81}$$

Applying the Implicit Function Theorem, we get that

$$\begin{aligned}
\frac{\partial X_t}{\partial p_t} &= -\frac{\partial \mathscr{G}/\partial p_t}{\partial \mathscr{G}/\partial X_t} &(82)\\
&= -\frac{-(1+r)}{-\frac{\partial E_t[\ell_{t+1}]}{\partial X_t}F - \frac{\partial E_t[\ell_{t+1}]}{\partial X_t}p_{t+1}} &(83)\\
&= -\frac{1+r}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}](F+p_{t+1})} &(84)\\
&= -\frac{1+r}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}](F+p_{t+1})} < 0. &(85)
\end{aligned}$$

Similarly, we can obtain

$$
\begin{aligned}
\frac{\partial X_t}{\partial p_{t+1}} &= -\frac{\partial \mathscr{G}/\partial p_{t+1}}{\partial \mathscr{G}/\partial X_t} && (86)\\
&= -\frac{1-E_t[\ell_{t+1}]}{-\frac{\partial E_t[\ell_{t+1}]}{\partial X_t}F - \frac{\partial E_t[\ell_{t+1}]}{\partial X_t}p_{t+1}} && (87)\\
&= \frac{1-E_t[\ell_{t+1}]}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}](F+p_{t+1})} && (88)\\
&= \frac{1-E_t[\ell_{t+1}]}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}](F+p_{t+1})} > 0. && (89)
\end{aligned}
$$

□

Proposition 2 (Smoothness at boundaries): *Stock controls can be initiated without letting the launch rate exceed the open access launch rate. Flow controls cannot be initiated without forcing the launch rate to exceed the open access launch rate.*

Proof. In both cases, I suppose that there is open access before the control is initiated. *Initiating a stock control:* Suppose a stock control is scheduled to take effect at t, that is, satellite owners in t begin paying p_t. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : \pi = rF + E_{t-1}[\ell_t]F + p_t. \tag{90}$$

Let the open access launch rate in $t-1$ with no stock control in t be $\hat{X}_{t-1} : \pi = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} > X_{t-1}$.

Initiating a flow control: Suppose a flow control is scheduled to be implemented at t, that is, satellite launchers in t begin paying p_t to launch. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : \pi = rF + E_{t-1}[\ell_t]F - (1-E_{t-1}[\ell_t])p_t. \tag{91}$$

Let the open access launch rate in $t-1$ with no flow control implemented in t be $\hat{X}_{t-1} : \pi = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} < X_{t-1}$.

□

Proposition 3 (Controlling the rate of deorbit): *Stock controls with positive prices can make satellite owners deorbit their satellites and induce net deorbits. Flow controls with positive prices cannot make satellite owners deorbit their satellites or induce net deorbits.*

Proof. A satellite owner facing a stock control in period t will deorbit if

$$p_t > \pi + (1-E_t[\ell_{t+1}])F - V^d. \tag{92}$$

The regulator can induce firms to deorbit in t by raising p_t high enough in $t-1$. A potential launcher in $t-1$ will not launch if

$$p_t > \pi + (1 - E_{t-1}[\ell_t])F. \tag{93}$$

By raising p_t high enough, the regulator can both discourage further launches and induce existing satellite owners to deorbit their satellites.

A satellite owner facing a flow control in period t will deorbit if

$$p_t : \pi + (1-\ell_t)\beta E_t[Q_{t+1}] < V^d, \text{ where } X_t : \beta E_t[Q_{t+1}] = F + p_t \tag{94}$$
$$\implies (1-\ell_t)(F+p_t) < V^d - \pi, \tag{95}$$

which cannot be satisfied by positive p_t, given $V^d < \pi$. A potential launcher in t will not launch if

$$p_{t+1} : \ \pi + (1 - E_t[\ell_{t+1}])p_{t+1} < rF + E_t[\ell_{t+1}]F + (1+r)p_t. \tag{96}$$

If $\pi < rF + E_t[\ell_{t+1}]F + (1+r)p_t$, equation 96 will not be satisfied for any positive p_{t+1}. Although equation 96 can be satisfied if p_{t+1} is sufficiently negative, this would require the regulator to commit to a path of ever-decreasing negative prices as long as they wished to prevent launches (as described earlier and in Lemma 7). Regardless, the regulator cannot induce net deorbits (no new arrivals and some deorbits) in $t+1$ with a positive p_{t+1}. □

Proposition 7 (ADR can reduce collision risk):

Proof. I show the result first for the introduction of debris removal services, then for the ongoing use of debris removal services.

The introduction of ADR: Suppose an active debris removal service will become available at date t. To clarify whether removal is an option or not, I explicitly include the conditioning variables in the loss rate, that is, $E_t[\ell_{t+1}]$ is written as $E_t[\ell_{t+1}|S_{t+1}, D_{t+1} - R_{t+1}]$ when removal is an option.

Under open access, firms without satellites at $t-2$ will launch until

$$E_{t-2}[\ell_{t-1}|S_{t-1}, D_{t-1}] = r_s - r. \tag{97}$$

At $t-1$, launchers will expect to be able to remove debris once their satellites reach orbit. They will launch until

$$E_{t-1}[\ell_t|S_t, D_t - R_t] = r_s - r - \frac{c_t}{F}R_{it}. \tag{98}$$

Comparing $E_{t-2}[\ell_{t-1}|S_{t-1},D_{t-1}]$ and $E_{t-1}[\ell_t|S_t,D_t-R_t]$ yields the necessary and sufficient conditions:

$$E_{t-2}[\ell_{t-1}|S_{t-1},D_{t-1}]-E_{t-1}[\ell_t|S_t,D_t-R_t]>0 \iff c_tR_{it}>0. \tag{99}$$

Ongoing use of ADR: Under open access, the equilibrium collision risk in $t+1$ after debris removal in t is

$$E_t[\ell_{t+1}]=r_s-r-\frac{c_{t+1}}{F}R_{it+1}.$$

Similarly, the equilibrium collision risk in t after debris removal in $t-1$ is

$$E_{t-1}[\ell_t]=r_s-r-\frac{c_t}{F}R_{it}.$$

Subtracting one equilibrium risk from the other yields the necessary and sufficient condition for ongoing debris removal to continue to reduce the collision risk:

$$\begin{aligned}E_{t-1}[\ell_t]-E_t[\ell_{t+1}]>0 &\iff r_s-r-\frac{c_t}{F}R_{it}-(r_s-r-\frac{c_{t+1}}{F}R_{it+1})>0\\ &\iff c_{t+1}R_{it+1}>c_tR_{it}.\end{aligned}$$

□

5.2 Technical assumptions and lemmas

Assumption 4. *Let S be a vector of state variables, and I_k be a vector of same size as S with 1 in the k^{th} position and 0 in all other positions. $\phi(\ell|S)$ is a conditional density which satisfies the following properties:*

1. *The derivative of $\phi(\ell|S)$ with respect to the k^{th} argument of S,*

$$\frac{\partial\phi(\ell|S)}{\partial S_k}=\lim_{h\to 0}\frac{\phi(\ell|S+I_kh)-\phi(\ell|S)}{h}\equiv\phi_S(\ell|S),$$

exists and is bounded $\forall\ell$, with $\phi_S(\ell|S)\neq 0$ for some ℓ, $\forall S$.

2. *Let S^1 and S^2 be two vectors which are identical except for the k^{th} entry, where $I_kS^1<I_kS^2$. $\forall S^1,S^2$, and $\forall A\in[0,1]$, define $\bar{\ell}_1\equiv\bar{\ell}(A,S^1),\bar{\ell}_2\equiv\bar{\ell}(A,S^2)$ such that*

$$\int_0^{\bar{\ell}_1}\phi_S(\ell|S^1)d\ell=\int_0^{\bar{\ell}_2}\phi_S(\ell|S^2)d\ell=A.$$

Then $\phi(\ell|S)$ satisfies a Lipschitz condition

$$\left|\left|\int_{\bar{\ell}_1}^{\bar{\ell}_2} \phi(\ell|S^2)d\ell\right|\right| < ||S^2 - S^1||,$$

The first condition places a lower bound on the change in collision probability from new satellite placements and ensures some smoothness for the changes in the density across the support.

The second condition places an upper bound on changes to the density $\phi(\ell|S,D)$ over the space of (S,D). The idea is that the additional area under the new density required to achieve a target area under the old density is bounded by the change in S or D. Physically, this requires that new satellites or debris will not be placed in orbits that will cause a drastic change in the collision probability. Rather, the change in collision probability from new launches should be bounded and proportional to the number of new satellites placed in orbit. Together, the physical implication of these two conditions is that new satellites or debris will cause some changes to the collision probability, but that those changes will be bounded across the possible outcomes. This is economically reasonable for satellites - a violation of this implies that firms are deliberately placing their satellites in risky orbits. This may be less reasonable for debris, since the orbits of debris objects resulting from collisions are uncontrolled and difficult to predict. These conditions facilitate the proofs of the lemmas below, but are not crucial to the main results of the paper.

Note that the proofs of the lemmas below often assume uniformly bounded functions. While no such property is proven for the value functions studied, realistic parameter choices should guarantee the existence of uniform bounds on the value functions.

Lemma 2. *(Measurable functions under changes in distribution) Let ℓ be a random variable with a conditional density $\phi(\ell|S)$ defined on the compact interval $[a,b]$ and with range $[r(a),r(b)]$. Let $f(\cdot):[r(a),r(b)] \to [f(a),f(b)]$ be a measurable function of ℓ. Then*

$$\int_a^b f(\ell)\frac{\partial \phi(\ell|S)}{\partial S}d\ell = \frac{\partial E[f(\ell)|S]}{\partial S}$$

Proof.

$$\begin{aligned}\int_a^b f(\ell)\frac{\partial\phi(\ell|S)}{\partial S}d\ell &= \int_a^b f(\ell)\lim_{h\to 0}\frac{\phi(\ell|S+h)-\phi(\ell|S)}{h}d\ell\\ &= \lim_{h\to 0}\frac{1}{h}\left(\int_a^b f(\ell)\phi(\ell|S+h)d\ell - \int_a^b f(\ell)\phi(\ell|S)d\ell\right)\\ &= \lim_{h\to 0}\frac{1}{h}\left(E[f(\ell)|S+h]-E[f(\ell)|S]\right)\\ &= \frac{\partial E[f(\ell)|S]}{\partial S}.\end{aligned}$$

□

Lemma 3. $\frac{\partial E[f(x)|S]}{\partial S} = 0\ \ \forall S$ *and* $\forall f(x)$ *which do not depend on* ℓ*, the argument of* $\phi(\ell|S)$.

Proof. From Assumption 4 and Lemma 2,

$$\begin{aligned}\frac{\partial E[f(x)|S]}{\partial S} &= \int_0^1 f(x)\left[\lim_{h\to 0}\frac{\phi(\ell|S+h)-\phi(\ell|S)}{h}\right]d\ell\\ &= f(x)\lim_{h\to 0}\frac{1}{h}\left[\int_0^1\phi(\ell|S+h)d\ell - \int_0^1\phi(\ell|S)d\ell\right]\\ &= f(x)\lim_{h\to 0}\frac{1}{h}[1-1] = 0.\end{aligned}$$

□

Lemma 4. *If* $f(\ell)$ *is a nonnegative and uniformly bounded function, then under assumption 1*

1. $\frac{\partial E[f(\ell)|S]}{\partial S} = 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} = 0\ \ \forall\ell$
2. $\frac{\partial E[f(\ell)|S]}{\partial S} < 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} < 0\ \ \forall\ell$
3. $\frac{\partial E[f(\ell)|S]}{\partial S} > 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} > 0\ \ \forall\ell$

Proof. For simplicity, the proof is written for a scalar-valued S. Extending the argument to vector-valued S is possible but not particularly informative.

The first statement, $\frac{\partial E[f(\ell)|S]}{\partial S} = 0$ if $\frac{\partial f(\ell)}{\partial \ell} = 0$, follows directly from Lemma 3 and the assumption that $f(\ell)$ is constant $\forall\ell$.

To show that $\frac{\partial E[f(\ell)|S]}{\partial S} < 0$ if $\frac{\partial f(\ell)}{\partial \ell} < 0\ \ \forall\ell$, without any loss of generality let $S^2 > S^1$. Pick $\bar{\ell}_1,\bar{\ell}_2 : \int_0^{\bar{\ell}_1}\phi_S(\ell|S^1)d\ell = \int_0^{\bar{\ell}_2}\phi_S(\ell|S^2)d\ell = A \in (0,1)$. Note that when $A = 0, \bar{\ell}_1 = \bar{\ell}_2 = 0$, and when $A = 1, \bar{\ell}_1 = \bar{\ell}_2 = 1,\ \ \forall S^1, S^2$. Assumption 1 implies that $\forall A \in (0,1)$, $\bar{\ell}_2 > \bar{\ell}_1$. Since

$\frac{\partial f(\ell)}{\partial \ell} < 0 \;\; \forall \ell,$

$$\begin{aligned}
&\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell > \int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell \\
\implies &\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell - \int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell > 0 \\
\implies &\int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell - \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell < 0 \\
\implies &\lim_{S^2 \to S^1} \frac{1}{S^2 - S^1}\left[\int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell - \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell\right] \\
&= \lim_{S^2 \to S^1} \frac{1}{S^2 - S^1}\left[\int_0^{\bar{\ell}_1} f(\ell)\{\phi_S(\ell|S^2)d\ell - \phi_S(\ell|S^1)\}d\ell + \int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\phi(\ell|S^2)d\ell\right] \\
&= \int_0^{\bar{\ell}_1} f(\ell) \lim_{S^2 \to S^1}\left\{\frac{\phi_S(\ell|S^2)d\ell - \phi_S(\ell|S^1)}{S^2 - S^1}\right\}d\ell + \lim_{S^2 \to S^1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell \\
&= \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell + \lim_{S^2 \to S^1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell < 0
\end{aligned}$$

By assumption 4 and $f(\ell) \geq 0$, $S^2 > S^1$, $\phi(\ell|S^2) \geq 0$,

$$\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell \geq 0.$$

Now, taking the limit as A goes to 1, we get

$$\lim_{A \to 1}\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell + \lim_{S^2 \to S^1}\lim_{A \to 1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell < 0,$$

where

$$\lim_{A \to 1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell = O(\bar{\ell}_2 - \bar{\ell}_1)$$

is a nonnegative remainder term is bounded by $\bar{\ell}_2 - \bar{\ell}_1$. This leaves us with

$$\lim_{A \to 1}\left[\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell\right] + \lim_{S^2 \to S^1} O(\bar{\ell}_2 - \bar{\ell}_1) < 0.$$

When $A = 1, \bar{\ell}_2 = \bar{\ell}_1 = 1$ and the remainder is exactly $0 \;\; \forall S^1, S^2$. Since S^1 was chosen arbitrarily, we can therefore say that

$$\int_0^1 f(\ell)\phi_S(\ell|S)d\ell \equiv \frac{\partial E[f(\ell)|S]}{\partial S} < 0 \text{ if } \frac{\partial f(\ell)}{\partial \ell} < 0 \;\; \forall \ell.$$

Repeating the argument above with $f(\ell)$ strictly increasing instead of decreasing yields the third statement, $\frac{\partial E[f(\ell)|S]}{\partial S} > 0$ if $\frac{\partial f(\ell)}{\partial \ell} > 0 \;\; \forall \ell$. □

6 Appendix B: Additional economic properties of active debris removal and open access orbit use

6.1 Incentives to cooperate in debris removal

Suppose all satellite owners agree to cooperate and individually purchase R^*_{it} units of debris removal. Owner i considers deviating and reducing her removal demands by $\varepsilon \in (0, R^*_{it}]$. Her payoff from not deviating by ε is

$$\underbrace{\pi - c_t R^*_{it} + (1 - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t])F}_{\text{Expected value of cooperating}} - [\underbrace{\pi - c_t(R^*_{it} - \varepsilon) + (1 - \tilde{E}_t[\ell_t | S_t, D_t - (R^*_t - \varepsilon)])F}_{\text{Expected value of deviating by } \varepsilon}]$$
$$= -\varepsilon c_t + [\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R^*_t] - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t]]F.$$

Cooperation is a strictly dominant Nash equilibrium if and only if

$$\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R^*_t] - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t] > \varepsilon \frac{c_t}{F} \quad \forall \varepsilon \in (0, R^*_{it}]. \tag{100}$$

Proposition 9 establishes an intuitive necessary and sufficient condition for cooperation to strictly dominate small deviations. If the change in the expected loss rate before removal is greater than the ratio of the cost of removal in t to the cost of launching a satellite in t, then a tiny deviation will cost a satellite owner more expected value through a lower survival rate than it will yield in a removal expenditure savings.

Proposition 9. *(Local stability of cooperation) Cooperation with any non-zero debris removal plan strictly dominates small deviations if the change in the equilibrium collision risk from another unit of debris is greater than the ratio of the removal price to the launch cost,*

$$\left.\frac{\partial \tilde{E}[\ell_t | S_t, \bar{D} - R_t]}{\partial \bar{D}}\right|_{\bar{D} = D_t} > \frac{c_t}{F}.$$

Proof. Cooperation with a non-zero debris removal plan is robust to all deviations ε for which

$$\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R^*_t] - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t] > \varepsilon \frac{c_t}{F}.$$

Cooperation with a non-zero debris removal plan strictly dominates small deviations if

$$\lim_{\varepsilon \to 0} \frac{\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R_t] - \tilde{E}_t[\ell_t | S_t, D_t - R_t]}{\varepsilon} > \frac{c_t}{F} \quad \forall R_t > 0$$
$$\implies \left.\frac{\partial \tilde{E}[\ell_t | S_t, \bar{D} - R_t]}{\partial \bar{D}}\right|_{\bar{D} = D_t} > \frac{c_t}{F} \quad \forall R_t > 0.$$

$\square$

The following proposition establishes that the debris removal solution described in equation 56 is in fact the cooperative private debris removal solution.

Proposition 10. *(A cooperative private removal plan) The debris removal solution described by equation 56 is maximizes the value of the currently-orbiting satellite fleet, given open access in that period.*

Proof. Given open access launch rates, the value of a satellite already in orbit is

$$Q_i(S,D) = \pi - cR_i + (1 - \tilde{E}[\ell])F.$$

Given open access launch rates, the value of all satellites already in orbit is

$$\begin{aligned} W(S,D) &= \int_0^S Q_i di \\ &= \pi S - cR + (1 - \tilde{E}[\ell])FS. \end{aligned}$$

Equation 56 is the first-order condition for the firm's problem,

$$Q_i(S,D) = \max_{0 \leq R_i \leq D/S} \{\pi - cR_i + (1 - \tilde{E}[\ell])F\}.$$

A constrained planner who maximizes the value of the currently-orbiting satellite fleet, taking open access to orbit as given, solves

$$\begin{aligned} W(S,D) &= \max_{0 \leq R \leq D} \{\pi S - cR + (1 - \tilde{E}[\ell])SF\} \\ &= \max_{\{0 \leq R_i \leq D/S\}_i} S\{\pi - cR_i + (1 - \tilde{E}[\ell])F\} \\ &= \max_{\{0 \leq R_i \leq D/S\}_i} SQ_i(S,D). \end{aligned}$$

The constrained planner's objective function is an individual satellite owner's objective scaled by the current size of the fleet, which the constrained planner takes as given. The individual removal solution given by equation 56 therefore characterizes a cooperative debris removal solution, where each firm behaves as an open-access-constrained social planner would command. □

6.2 Nonconvexities and corner solutions

For brevity, I write $E_t[\ell_{t+1}]$ as $L(S_t, D_t - S_t R_{it})$ in this subsection and use S and D subscripts to indicate the respective partial derivatives. Since these results are intratemporal in nature, I also drop time subscripts.

Upper bounds on damages, nonconvex stock decay rates, and complementarities between stocks in damage production each imply nonconvexities in the marginal benefits of abatement. The static private marginal benefits from abatement reflect two of these features:

1. at most all satellites can be destroyed in collisions, implying the upper bound on the rate of satellite-destroying collisions;
2. the marginal effect of debris on the number of satellite-destroying collisions depends on the number of satellites in orbit, which makes collisional complementarity or substitutability between satellites and debris possible.

When the satellite and debris couplings in the collision rate depend on each other, that is, $L_{SD} \neq 0$, changes in the satellite stock can change the returns to scale for debris removal. The dynamic benefits of debris abatement also include the effect of fragment growth from collisions between debris. This effect implies that the net marginal rate of debris decay $(\delta - G_D(S, D-R))$ can be negative.

The marginal benefit of removal is the private value of reducing the probability of a satellite-destroying collision. Debris removal has diminishing marginal benefits if and only if the collision rate is strictly convex in debris. The upper bound on $L(S,D)$ implies that debris removal will have increasing marginal benefits when the risk of a collision gets high enough. Figure 11 shows two examples of this, one with a negative exponential collision rate (globally concave) and another with a sigmoid collision rate (first convex and later concave).

[Figure 11 about here.]

For any positive initial level of debris and satellites (S,D), removal must be nonnegative and no more than all of the debris can be removed. When all satellite owners are identical, the maximum that any one can remove is D/S. This closes the feasible set. Any intermediate amount can also be removed, making the feasible set convex.

The nonconvexity of marginal removal benefits complicates analysis of the optimal amount of removal. There are two cases: the collision rate is globally concave, or the collision rate is convex over some nonnegative interval.

1. If the collision rate is globally concave, there can be no interior solution to the satellite owner's removal problem. Global concavity implies increasing marginal benefits of debris removal, so satellite owners will choose either to remove all debris or none of it.
2. If the collision rate is convex over some nonnegative interval, an interior solution is possible but not guaranteed. For example, suppose the collision rate is convex initially and concave near the end, as in the sigmoid case in Figure 11. Either the right-most

intersection of marginal benefits and marginal costs is optimal (where equation 56 and inequality 57 hold) or else zero removal is optimal.

Determining which corner is optimal when the collision rate is globally concave case is straightforward. If the profits of full removal are greater than the profits of zero removal, full removal is optimal; if not, zero removal is optimal.

With local convexities, the problem is more complicated. One approach is as follows. First, select all solutions to the removal first-order condition (equation 56) which satisfy the second-order condition (inequality 57), and include them in a set with zero removal and full removal. This is the set of candidate solutions. Calculate the profits of each candidate solution, and select the one with the highest profits. This procedure is computationally tractable over a closed and convex support as long as the collision rate function is reasonably well-behaved. Figure 12 illustrates how nonconvexity of the collision rate affects profits and the optimal level of removal.

[Figure 12 about here.]

6.3 Comparative statics of cooperative debris removal and open access launching

I show three results about the demand for debris removal in this section.

First, there is a unique cooperatively-optimal post-removal level of debris for any given level of the satellite stock. This is a consequence of the linear cost (to satellite owners) of debris removal and the monotonicity of the expected collision risk in debris. Due to the linearity, cooperative satellite owners will pursue a most-rapid approach path to the optimal post-removal level of debris in every period. Were the cost nonlinear, the most-rapid approach path would no longer be optimal but the optimal level of debris would remain unique due to monotonicity.

Second, if satellites and debris are "strong enough" complements in producing collision risk, increasing the number of satellite owners in orbit will reduce the optimal post-removal level of debris. This spillover effect in debris removal suggests that a "dynamic virtuous cycle" of active debris removal may be possible: removal in one period can spur entry in the next, which in turn spurs more removal in the following period. Although the functional forms I use rule this effect out, those forms are simplified from a statistical mechanics approximation of orbital interactions. A higher-fidelity model may allow this possibility. A static analog of this effect can be seen in Figure 9.

Third, the open access launch rate may be *increasing* in the launch cost. Though this result seems counterintuitive, it is a natural consequence of three features of open access orbit use:

1. open access drives the value of a satellite down to the launch cost;
2. the amount of removal is increasing in the launch cost;
3. new entry can reduce the individual expenditure required from cooperative firms to achieve the optimal post-removal level of debris.

The cooperative cost-savings from new entry exceeding the effect of new entry on collision risk is necessary and sufficient for the open access launch rate to be increasing in the launch cost.

Together, these results suggest that the use of debris removal can result in interesting and counterintuitive dynamics in orbit use. Though these results are relevant to understanding the effects of debris removal technologies on orbit use, I omit their proofs from this section. Interested readers may find the proofs in the Appendix, section 5.

Cooperative private debris removal:

Lemma 5. *(Law of cooperative private debris removal demand) The cooperative private debris removal demand is*

1. *weakly decreasing in the price of removing a unit of debris, and*
2. *weakly increasing in the cost of launching a satellite.*

Proof. I consider corner solutions first, then interior solutions. I characterize how interior solutions change in response to a change in the removal price, then show that increases in the price can only induce the firm to reduce their removal demands even at corners. I refer to the non-optimized value of a satellite as $Q_i(R_i)$.

The full removal corner: The first part of the proposition is trivially true at the full removal corner, since the amount of debris removal purchased cannot increase at this corner. So it must either stay the same, or decrease, in response to an increase in the price of removal. For the second part, suppose a firm initially finds full removal optimal. Reducing the amount of debris by a positive amount ε in response to a change in launch cost removed is optimal if and only if, at the new launch cost,

$$\begin{aligned}
& Q_i(D/S-\varepsilon)-Q_i(D/S)>0 \;\; \forall \varepsilon \in \left(0, \frac{D}{S}\right] \\
\implies & \pi + F - c\frac{D}{S} + c\varepsilon - \tilde{E}[\ell|S, D-S(D/S-\varepsilon)]F - \pi - F + c\frac{D}{S} + \tilde{E}[\ell|S,0]F > 0 \\
\implies & c\varepsilon - (\tilde{E}[\ell|S,\varepsilon] - \tilde{E}[\ell|S,0])F > 0 \\
\implies & \frac{\tilde{E}[\ell|S,\varepsilon] - \tilde{E}[\ell|S,0]}{\varepsilon} < \frac{c}{F} \;\; \forall \varepsilon \in \left(0, \frac{D}{S}\right].
\end{aligned}$$

If full removal was optimal to begin with, then an increase in the launch cost cannot make it optimal to switch strategies. The above inequality also shows how an increase in the cost of removal can induce a firm to reduce the amount of removal purchased.

The zero removal corner: Consider the profits from increasing the amount of removal from zero to ε in response to a change in the launch cost or removal price. The change is privately optimal if and only if, at the new cost or price,

$$\begin{aligned}
& Q_i(\varepsilon)-Q_i(0)>0 \;\; \forall \varepsilon \in \left(0, \frac{D}{S}\right] \\
\implies & \pi + F - c\varepsilon - \tilde{E}[\ell|S, D-S\varepsilon]F - \pi - F + \tilde{E}[\ell|S,D]F > 0 \\
\implies & -c\varepsilon - [\tilde{E}[\ell|S,D-S\varepsilon] - \tilde{E}[\ell|S,D]]F > 0 \\
& \frac{\tilde{E}[\ell|S,D-S\varepsilon] - \tilde{E}[\ell|S,D]}{\varepsilon} > \frac{c}{F} \;\; \forall \varepsilon \in \left(0, \frac{D}{S}\right].
\end{aligned}$$

If zero removal was optimal to begin with, then an increase in the price of removal cannot make it optimal to switch strategies. An increase in the cost of launching a satellite, however, may induce a firm to begin removing debris.

For interior solutions: From equation 56,

$$R_i t : \mathscr{H} = c - \frac{\partial \tilde{E}[\ell]}{\partial D} SF = 0. \tag{101}$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}
\frac{\partial R_i}{\partial c} &= -\frac{\partial \mathscr{H}/\partial c}{\partial \mathscr{H}/\partial R_i} \\
&= -\frac{1}{\frac{\partial^2 \tilde{E}[\ell]}{\partial D^2} S^2 F} < 0.
\end{aligned}$$

Strict negativity follows from the second order condition (inequality 57). If there are multiple solutions and the removal price increase causes firms to jump from interior one solution to another, they must jump to a solution with less removal.

Similarly, from applying the Implicit Function Theorem to $\mathscr{H}$,

$$\frac{\partial R_i}{\partial F} = -\frac{\partial \mathscr{H}/\partial D}{\partial \mathscr{H}/\partial R_i}$$
$$= \frac{\frac{\partial \tilde{E}[\ell]}{\partial D} S}{\frac{\partial^2 \tilde{E}[\ell]}{\partial D^2} S^2 F} > 0.$$

Strict positivity follows from the second order condition (inequality 57). If there are multiple solutions and the launch cost increase causes firms to jump from interior one solution to another, they must jump to a solution with more removal. □

The intuition for this result is simple. Satellite owners pay for debris removal. When the price of removal rises, the demand for removal falls. Under open access the continuation value of a satellite is the cost of launching. So, the demand for debris removal increases when satellites become more valuable. Figure 13 illustrates Lemma 5.

[Figure 13 about here.]

What about changes in the satellite and debris stocks? Increases in the debris stock increase the cost of achieving any given level of reductions, but may also increase the marginal benefit of removal if the collision rate is locally concave. Increases in the satellite stock may increase the marginal benefit of removal if the collision rate is locally jointly concave, but also increase the number of firms in the market for removal and give existing firms an incentive to reduce their expenditures. I examine this question in Propositions 11 and 12.

Proposition 11. *(Cooperative demand for debris removal and the state of the orbit) The optimal post-removal debris level is independent of the pre-removal debris level, but depends on the number of satellites in orbit.*

Proof. Proof. I focus on interior solutions, but the result follows for corner solutions as well due to the monotonicity of $\tilde{E}_t[\ell_t]$ in S_t and D_t.

From equation 56,

$$R_{it} : \mathscr{H} = c_t - \frac{\partial \tilde{E}_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} S_t F = 0.$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\frac{\partial R_{it}}{\partial D_t} = -\frac{\partial \mathscr{H}/\partial D_t}{\partial \mathscr{H}/\partial R_{it}}$$
$$= \frac{1}{S_t} > 0.$$

The total quantity of debris removed is

$$R_t = \int_0^{S_t} R_{it} di = S_t R_{it}.$$

Suppose that a positive amount of removal is optimal before and after the change in debris. Differentiating R_t with respect to D_t and using the earlier results for individual removal demands,

$$\frac{\partial R_t}{\partial D_t} = S_t \frac{\partial R_{it}}{\partial D_t} = 1.$$

The monotonicity of $\tilde{E}_t[\ell_t]$ in S_t and D_t also implies that, for any S_t, there is a unique $D_t - R_t$ such that

$$c_t = \frac{\partial \tilde{E}_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} S_t F.$$

□

□

Proposition 11 shows that when it is optimal to remove debris, firms will increase their removal efforts in response to increases in debris. The uniqueness of the optimal post-removal debris stock requires aggregate removal demanded to match changes in the debris stock. This is analogous to the uniqueness of optimal escapement policies in fisheries management. Figure 14 illustrates this behavior.

[Figure 14 about here.]

Proposition 12. *Additional satellite owners decrease the cooperative individual debris removal demand unless satellites and debris are "strong enough" complements in collision risk production.*

Proof. I show the result for interior solutions. A similar condition also holds for corner solutions.

The total quantity of debris removed is

$$R_t = \int_0^{S_t} R_{it} di = S_t R_{it}.$$

Differentiating R_t with respect to S_t,

$$\frac{\partial R_t}{\partial S_t} = R_{it} + S_t \frac{\partial R_{it}}{\partial S_t}.$$

R_{it} and S_t are both nonnegative by definition. It follows that

$$\frac{\partial R_t}{\partial S_t} > 0 \iff \frac{R_{it}}{S_t} > -\frac{\partial R_{it}}{\partial S_t}.$$

This is always true when individual removal demands increase in response to additional satellite owners ($\frac{\partial R_{it}}{\partial S_t} > 0$). The following steps establish the complementarity condition for interior solutions.

From equation 56,

$$R_{it} : \mathscr{H} = c_t - \frac{\partial \tilde{E}_t[\ell_t|S_t, D_t - R_t]}{\partial D_t} S_t F = 0.$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}\frac{\partial R_{it}}{\partial S_t} &= -\frac{\partial \mathscr{H}/\partial S_t}{\partial \mathscr{H}/\partial R_{it}}\\ &= \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} - \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_{it}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t} \lesseqgtr 0.\end{aligned}$$

So, increases in the amount of debris must increase the privately optimal amount of removal at all interior solutions, while increases in the number of satellites will have ambiguous effects. The privately optimal demand for removal will be increasing in the number of satellites if and only if

$$\begin{aligned}\frac{\partial R_{it}}{\partial S_t} > 0 &\iff \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} - \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > 0\\ &\iff \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > -\frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S^2} + \frac{R_i}{S}\\ &\iff \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} < -\frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{S} + \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i.\end{aligned}$$

The right hand side of the final line is strictly negative at an interior optimum from the second-order condition, inequality 57. So $\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t}$ must be "sufficiently" negative for the presence of new satellites to increase privately optimal removal. Economically, this means that satellites and debris are "strong enough" substitutes.

□

Proposition 12 shows that firms may increase or decrease their removal demands in response to more firms entering the orbit. At the full removal corner, they will always reduce their demands. This is a cooperative cost-sharing effect: it remains optimal to remove all debris, but the contribution required of each firm decreases when more firms enter. At an interior solution, their response to more satellites depends on two effects: a congestion effect and a cooperation effect. Their net effect depends on the collision rate's convexity in debris and satellites, particularly whether satellites and debris are "strong enough" complements in producing collision risk. If

- the cooperation and congestion effects are collectively positive, then the presence of more satellites increases the marginal benefit of debris removal (positive spillover effects);
- the cooperation and congestion effects are collectively negative, then the presence of more satellites decreases the marginal benefit of debris removal (negative spillover effects).

These shifts are shown algebraically in the Appendix, section 9.4. Conditional on a positive amount of removal being optimal, increases in debris are matched by increases in removal. Increases in the number of satellites can result in aggregate removal decreases if the individual demand reduction is large enough. Given the functional forms I assume for simulations, the shifts are always collectively negative. I remain agnostic about the "correct" functional form to assume.

Any increase in debris is matched by a commensurate increase in aggregate removal. Since satellites are identical, owners collectively agree on the optimal level of debris. The total quantity of debris removed can not be decreasing in the number of satellites unless individual owners reduce their removal demands in response to more satellites in orbit. Proposition 12 shows this is a necessary but not sufficient condition.

Proposition 11 also offers some insight into the effects of launches on privately optimal removal. With no launch debris, the effect of a marginal launch on privately optimal removal is only the effect of a new satellite on privately optimal removal. With launch debris, the effect of a launch is a combination of the effect of a new satellite and the effect of new debris.

Open access launching: The option - or, in the cooperative case studied here, the obligation - to remove space debris alters the incentives of open access satellite launchers. While they will still launch until expected profits are zero, the expected collision risk is no longer the only object which equilibriates their launching behavior. In addition to expected collision risk, the expenditure they expect to incur removing debris as satellite owners will also adjust to equilibriate the launch rate. Though they will be price takers when their satellites reach orbit, they can anticipate the number of satellite owners who will contribute to debris removal. This acts in the opposite direction as the expected collision risk: while more satellites in orbit increases risk, more firms with satellites in orbit decreases each individual firm's debris removal expense. Since debris removal also reduces collision risk, the net effect of introducing debris removal financed by satellite owners may be more launches than would otherwise occur.

Indeed, this is precisely what occurs in the cases simulated here.

In addition to this perhaps-counterintuitive effect, it is plausible that an increase in the cost of launching a satellite could increase the launch rate. This is not as pathological a case as it may seem at first. Since open access drives the value of a satellite down to the launch cost, and the cooperatively-optimal amount of debris removal which satellite owners will pay for is increasing in the launch cost, and increase in the launch cost under open access could increase the value of owning a satellite by more than it increases the cost of launching it, at least locally near an existing equilibrium. This is not a violation of the law of demand for satellite ownership; rather, it is a violation of the "all else equal" clause. Assumption 5 describes a necessary and sufficient condition to rule this case out.

Assumption 5. *(New launches reduce the expected profits of satellite ownership) The change in individual removal expenses from a marginal satellite launch is smaller in magnitude than the sum of the change in expected future collision costs from a marginal satellite launch and the change in individual removal expenses from a marginal piece of launch debris. Formally,*

$$\left|\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F+m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F+\frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right)\right|>\left|\frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}\right|.$$

If this assumption is violated, then launches increase the profitability of owning a satellite through the debris removal expenditure channel described above. It is also possible that increases in the cost to satellite owners of removing a unit of debris could increase the launch rate Assumption 6 describes an additional condition necessary for increases in the price of debris removal to reduce the launch rate. An increase in the price of debris removal will reduce the cooperatively-optimal amount of debris removal satellite owners purchase, potentially reducing the total debris removal expenditure and increasing the profits of owning a satellite. As in the case of launch rates being increasing in launch costs, this is not a violation of the law of demand for satellite ownership; it is a violation of the "all else equal" clause.

Assumption 6. *(Removal expenditure is increasing in the removal cost) The cooperative private debris removal expenditure is increasing in the price of removing a unit of debris. Formally,*

$$\frac{\partial}{\partial c_{t+1}}(R_{it+1}c_{t+1})=R_{it+1}+\frac{\partial R_{it+1}}{\partial c_{t+1}}c_{t+1}>0.$$

Assumption 6 states that the amount of debris removed (R_{it+1}) is larger than the reduction in removal due to a price increase ($\frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}$, which is weakly negative from Proposition 5). This is likely to hold whenever the change in individual removal demands from a change in removal cost is small, for example,if removal demand is in the interior before and after the change. It is unlikely to hold if the opposite is true, for example,if the change in removal cost causes individual removal demands to jump from the full removal corner to the zero removal corner at a time when there are few satellites and many debris fragments. Though future cooperative private debris removal demands are an anticipated cost to current satellite launchers, those same launchers may find their willingness to launch increasing in the cost of removal if it reduces the burden of cooperating and purchasing removal.

Proposition 13. *(Private demand for satellite ownership) The open access launch rate is*

1. *strictly decreasing in the cost of launching a satellite if and only if new launches reduce the expected profits of satellite ownership; and*
2. *strictly decreasing in the price of removing a unit of debris in $t+1$ only if new launches reduce the expected profits of satellite ownership AND the cooperative private expenditure on debris removal is increasing in the cost of removal.*

Proof. From equation 54,

$$X_t : \mathscr{F} = \pi - rF - E_t[\ell_{t+1}]F - R_{it+1}c_{t+1} = 0.$$

Applying the Implicit Function Theorem to $\mathscr{F}$ and assuming R_{it+1} is chosen optimally (as described in Proposition 5),

$$\begin{aligned}\frac{\partial X_t}{\partial F} &= -\frac{\partial \mathscr{F}/\partial F}{\partial \mathscr{F} \partial X_t} \\ &= -\frac{r + E_t[\ell_{t+1}] + \frac{\partial R_{it+1}}{\partial F}c_{t+1}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right) + \frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}},\end{aligned}$$

which is negative for all parameter values when Assumption 5 holds.

Similar manipulations yield

$$\begin{aligned}\frac{\partial X_t}{\partial c_{t+1}} &= -\frac{\partial \mathscr{F}/\partial c_{t+1}}{\partial \mathscr{F} \partial X_t} \\ &= -\frac{R_{it+1} + \frac{\partial R_{it+1}}{\partial c_{t+1}}c_{t+1}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right) + \frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}}.\end{aligned}$$

$\frac{\partial X_t}{\partial c_{t+1}}$ is negative only if Assumptions 5 and 6 hold. □

Given Assumption 5, Assumption 6 is necessary and sufficient for $\frac{\partial X_t}{\partial c_{t+1}}$ to be negative. A simultaneous violation of Assumptions 5 and 6 would indicate that the private marginal cost of orbit use was decreasing in the costs of access and debris removal - a counterintuitive situation, but not an *a priori* impossible one.

7 Tables and figures

Figures in the main text

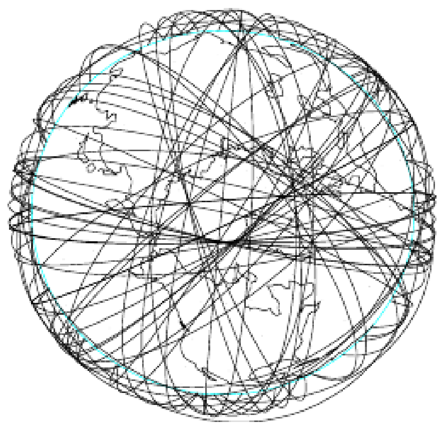

Figure 1: *Orbits of 56 cataloged satellites with mean altitudes between 700-710km. Source: Johnson (2004).*

Table 1: Currently-operational satellites by origin, orbit class, and orbit type as of April 30, 2018

Breakdown of operating satellites					**Total**
by Country of origin	United States: 859	Russia: 146	China: 250	Other: 631	1,886
by Orbit Class	LEO: 1,186	MEO: 112	Elliptical: 40	GEO: 548	1,886
Breakdown of US satellites by Owner Type	Civil: 20	Commercial: 495	Government: 178	Military: 166	859

Source: Union of Concerned Scientists (2018).

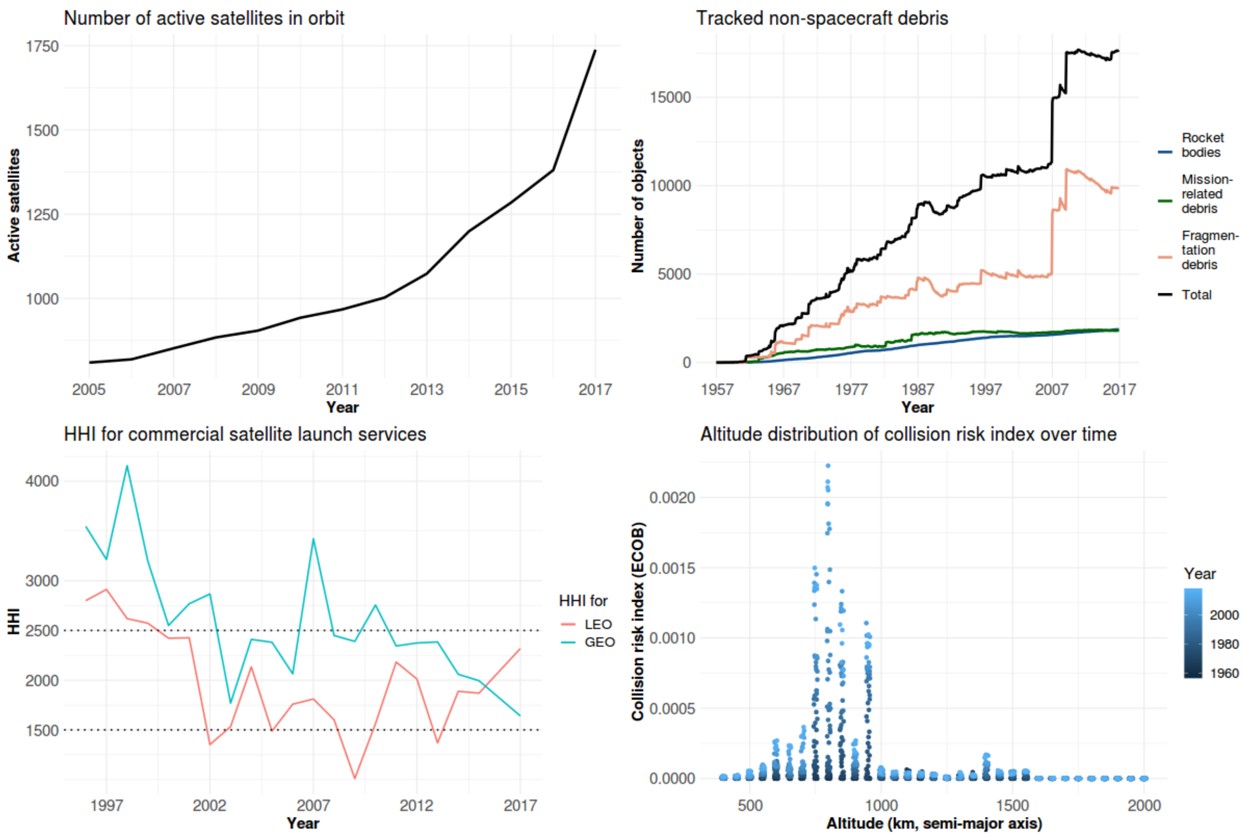

Figure 2: *Trends in orbit use.*

Upper left panel: Number of active satellites in orbit per year since 2005.

Upper right panel: Monthly tracked non-spacecraft debris. These do not include derelict satellites which were not deorbited.

Lower left panel: Herfindahl-Hirschman Index for commercial launch services to low-Earth orbit and geostationary orbit.

Lower right panel: Evolution over time of the spatial distribution of ECOB collision risk index in low-Earth orbit. The large spike between 500-1000km is driven by a combination of commercial activity and China's 2007 anti-satellite missile test.

Sources: Union of Concerned Scientists (2018), NASA Orbital Debris Program Office (2017), and Letizia, Lemmens, and Krag (2018).

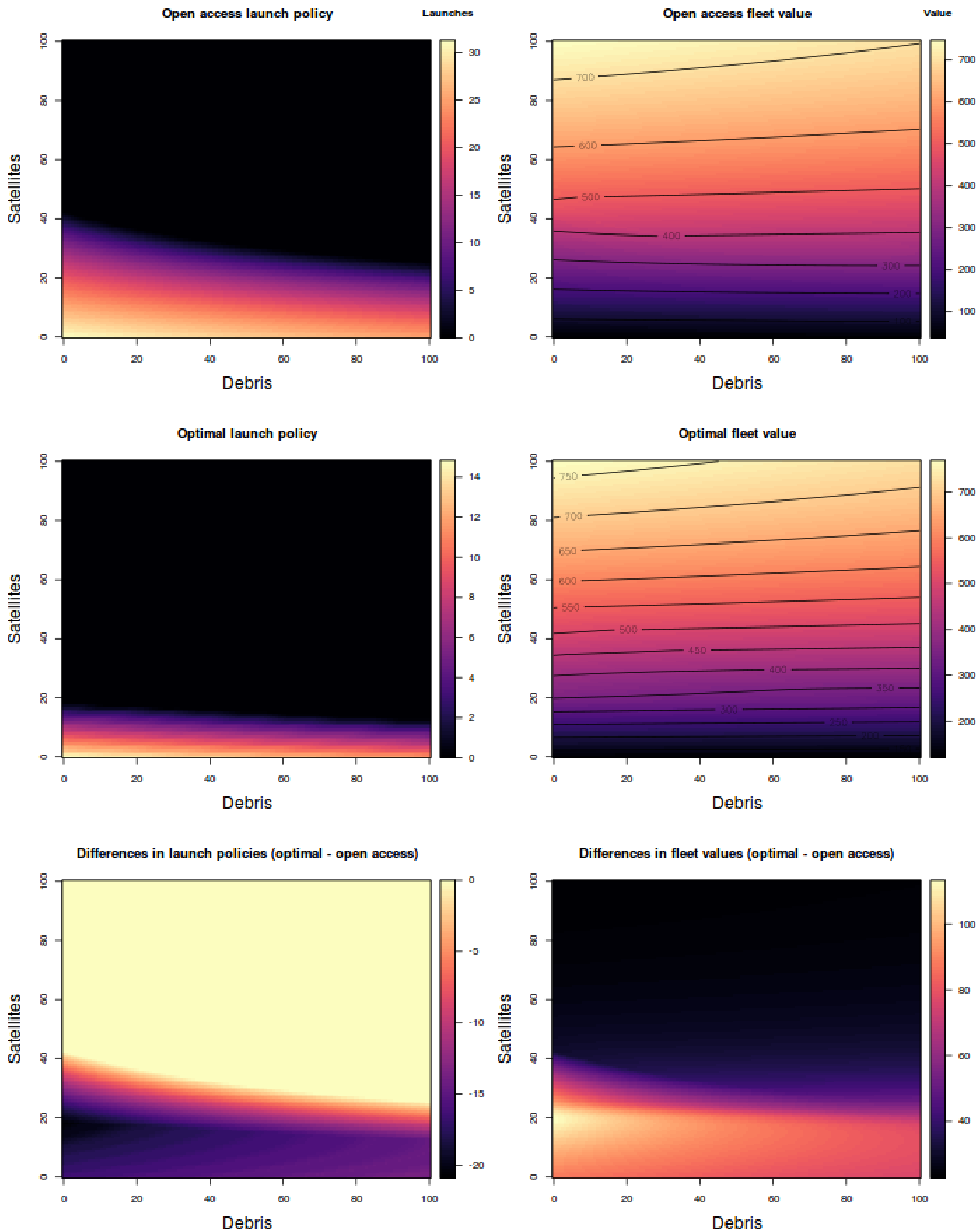

Figure 3: *An example of the gap between open access and optimal launch policies, with the corresponding gap in fleet values.*
The planner launches fewer satellites in every state than open access firms would. The value gap is maximized when (a) there is no debris and (b) the planner would stop launching satellites but open access firms do not.

Table 2: Examples of different types of orbit management policies

	Quantity control	**Price control**
Flow control	Launch permits	Launch taxes
Stock control	Orbit use leases	Satellite taxes

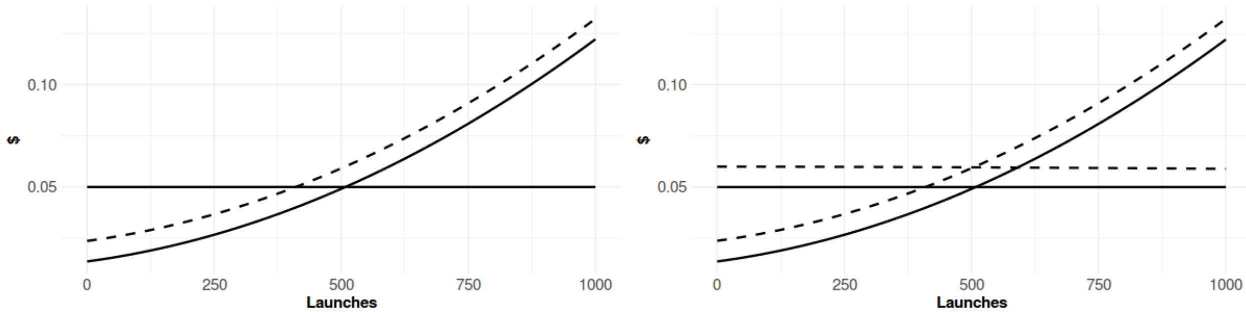

Figure 4: *Private marginal benefits and costs of launching a satellite under stock (left) and flow (right) controls.*

Left panel: The horizontal solid line is the marginal benefit of launching a satellite, while the upward-sloping solid line is the marginal cost. The dashed line indicates the effect of imposing a stock control: the marginal cost is increased, lowering the equilibrium number of satellites launched.

Right panel: The horizontal solid line is the marginal benefit of launching a satellite, while the upward-sloping solid line is the marginal cost. The dashed lines indicate the effects of imposing a flow control: the period t control raises the marginal cost of launching in t, but the entry restriction of the period $t+1$ control raises the marginal benefit of launching in t. Corollary 1 establishes that a constant price will increase the marginal cost by more than it increases the marginal benefit, but the net effect size may be very small.

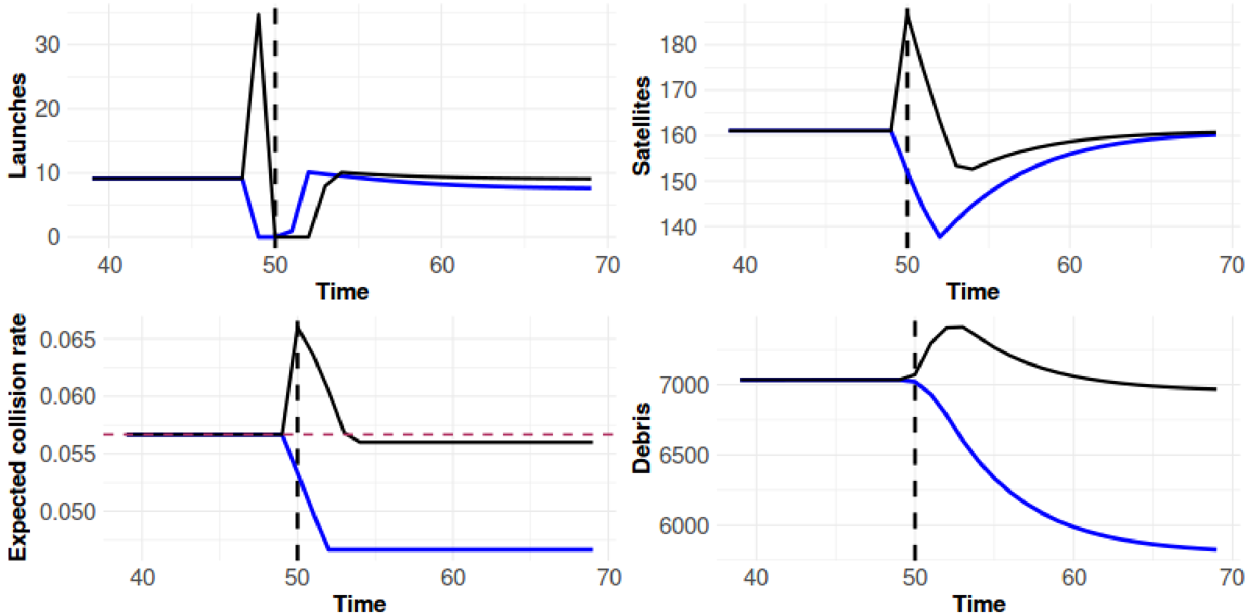

Figure 5: *The effects of introducing a generic constant stock (blue line) or flow control (black line).* The purple dashed line shows the equilibrium collision risk under open access. Introducing a stock control smoothly reduces the expected collision risk and debris stock, while introducing a flow control forces both to jump above the open access levels before they are reduced. The assumption of constancy is made for exposition, and is not important to the result.

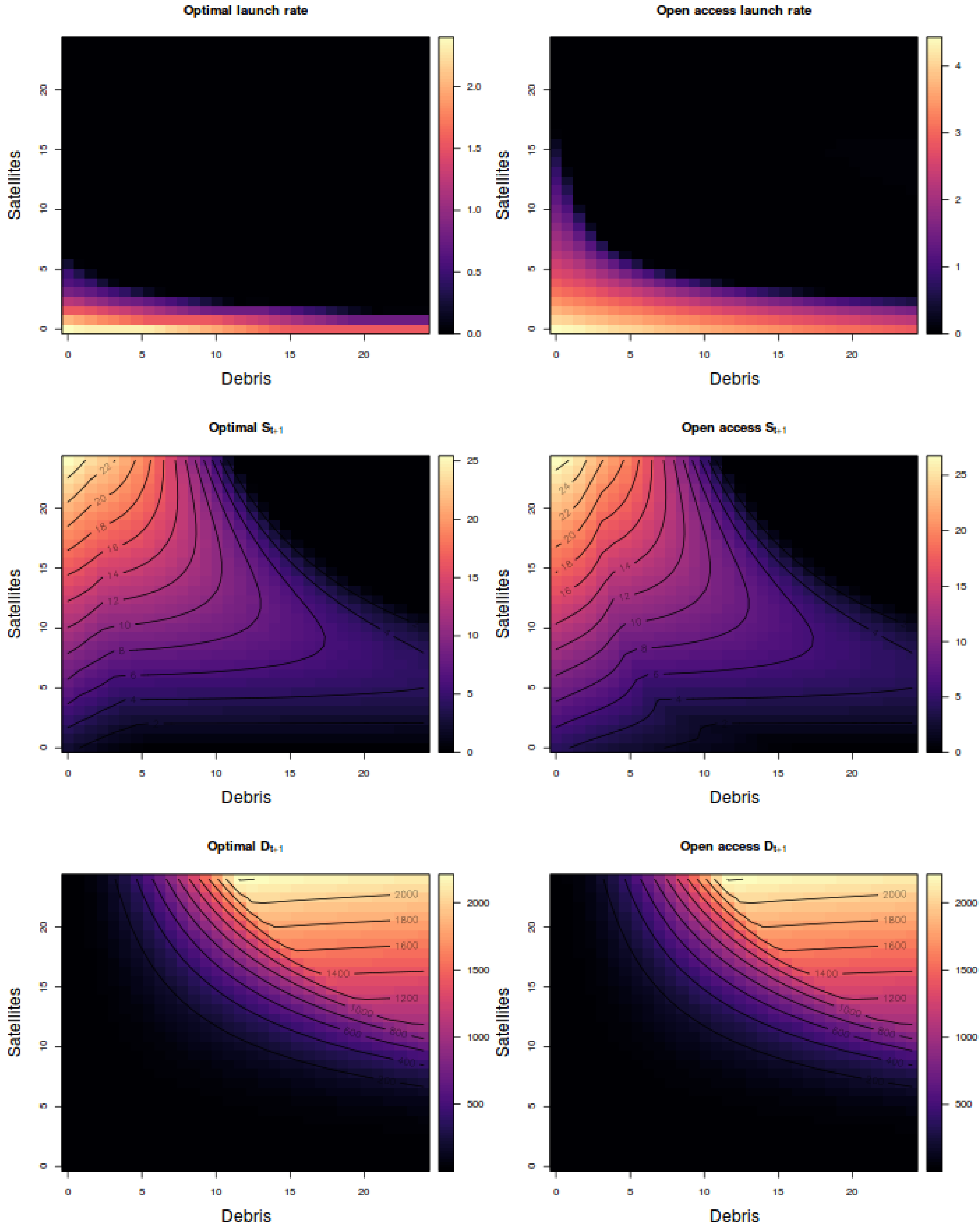

Figure 6: *Optimal and open access stocks and launch rates.*
Left column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
Right column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
The per-period return on a satellite is normalized to 1, the discount factor is set to 0.95, and the launch cost is set to 10. The open access next-period satellite stock is small but not zero in the upper right of the figure, while the optimal next-period satellite stock is zero there.

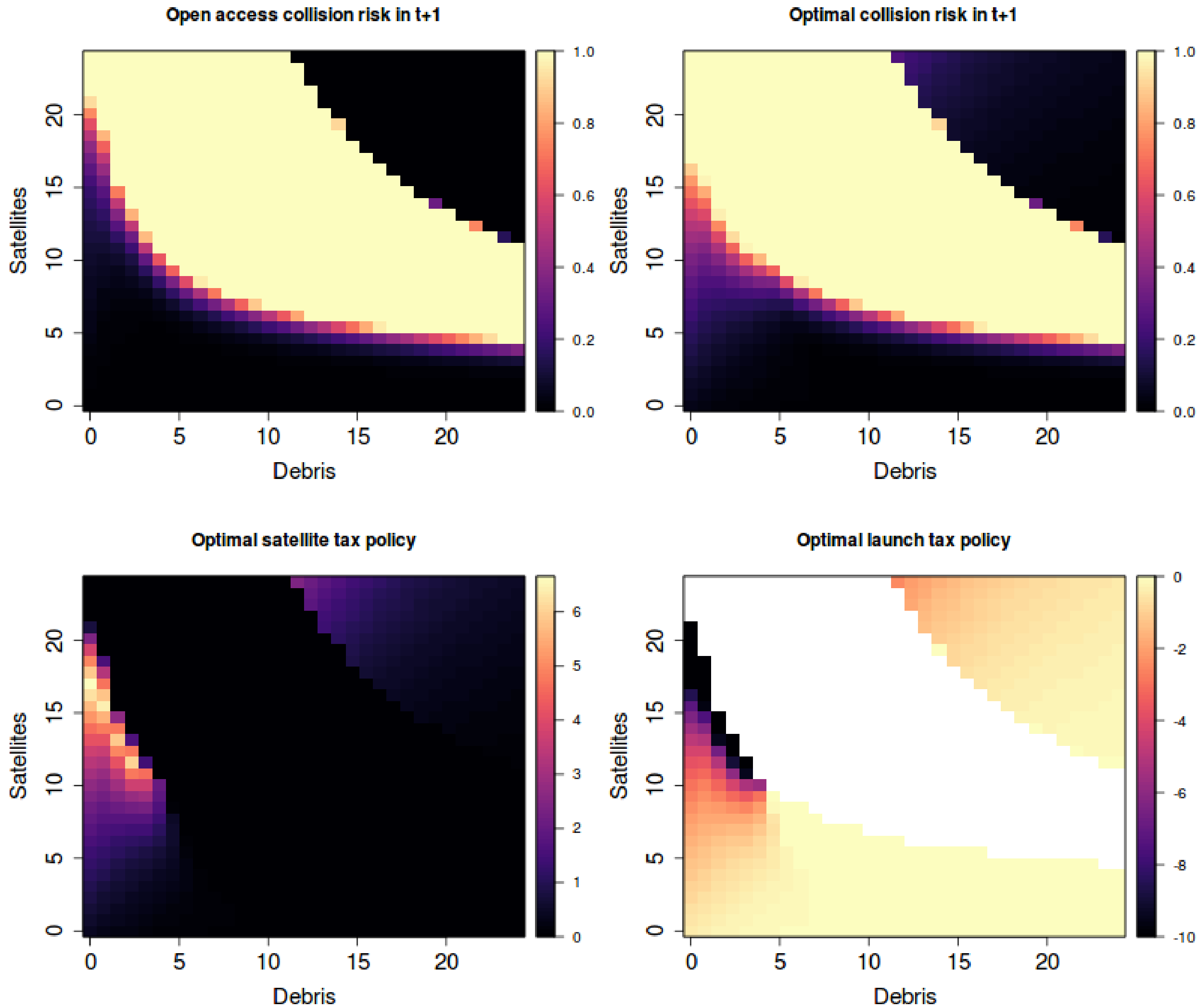

Figure 7: *Optimal space traffic control policies.*
Upper panels: The collision risk in $t+1$ ($E_t[\ell_{t+1}]$) under the optimal launch plan (left) and open access launch plan (right).
Lower left panel: An optimal satellite tax (stock control) in $t+1$.
Lower right panel: An optimal launch tax (flow control) in $t+1$. The tax in period t is normalized to 0.
The tax rates should be read as multiples of the per-period satellite return (normalized to 1). The white areas in the launch tax are where the collision risk is 1 and the tax is undefined; see Lemma 8 for an explanation of this feature. The collision risk jumps from 1 to 0 in the upper right section of the figures because there are no satellite left to be destroyed; see Figure 6 for the underlying satellite and debris stocks and launch rates. The marginal external cost is computed as $E_t[\xi(S_{t+1}, D_{t+1})] = E_t[\ell_{t+1}|\text{open access}] - E_t[\ell_{t+1}|\text{optimal}]$, following equations 11 and 15. The tax rates are then computed according to equations 33 and 34. The jump in the tax rates in the upper right is due to the slight gap in the satellite stocks described in Figure 6.

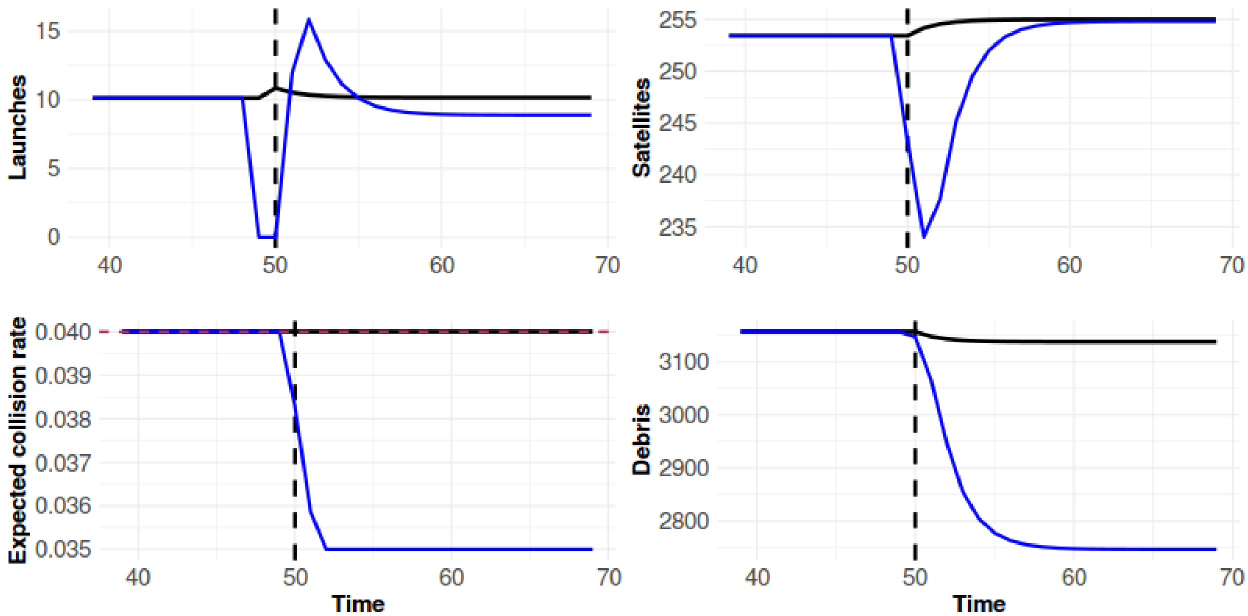

Figure 8: *The effects of exogenous removal for free (black line) or a mandatory fee (blue line). When debris removal is provided to satellites owners for free, potential launchers respond by launching more satellites - even though the debris stock falls, the equilibrium collision risk remains unchanged. The equilibrium collision risk will fall when active debris removal is an option if and only if it is costly to satellite owners. In the case of costly debris removal, the launch rate falls to zero until the expected collision risk is no longer above the new equilibrium level. The dashed red line shows the equilibrium collision risk under open access.*

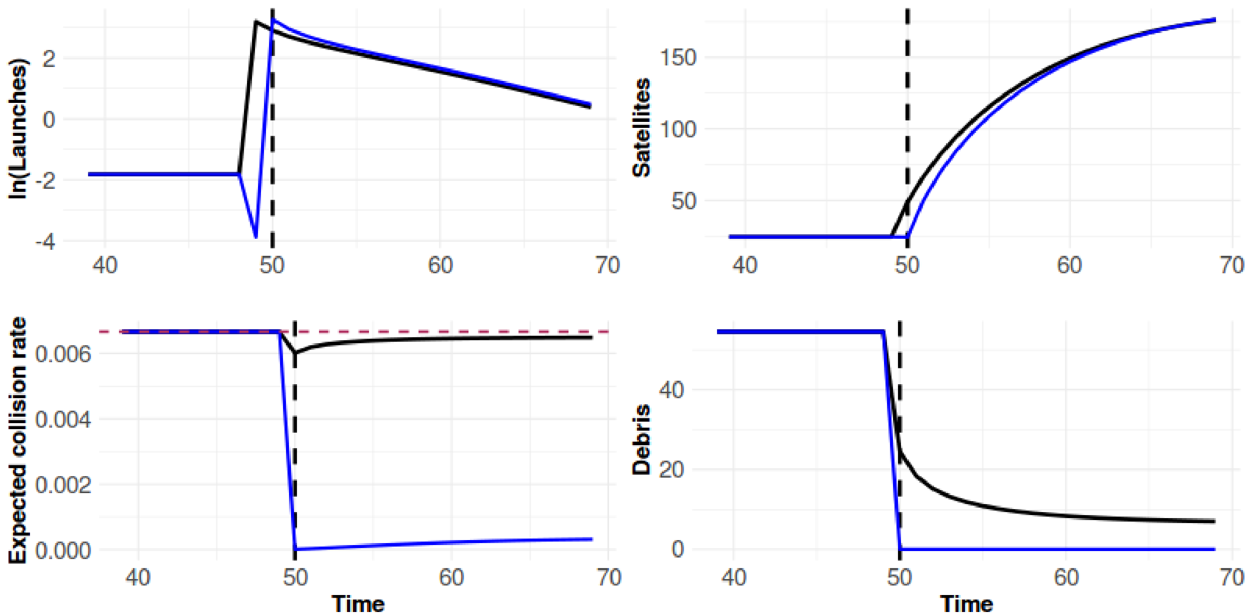

Figure 9: *The effects of endogenously chosen cooperative debris removal (blue line) and exogenous removal for a mandatory fee (black line). The exogenous removal path in the exogenous case is set equal to endogenous removal path. Endogenous removal reduces both the equilibrium collision risk and the debris stock more effectively than exogenous removal, even if the same removal schedule is used. The endogenous removal schedule and launch response involves completely cleaning the orbit initially, and keeping the orbit relatively clean after. The same removal schedule provided exogenously induces firms to launch earlier than they would if they chose the schedule. The dashed red line shows the equilibrium collision risk under open access.*

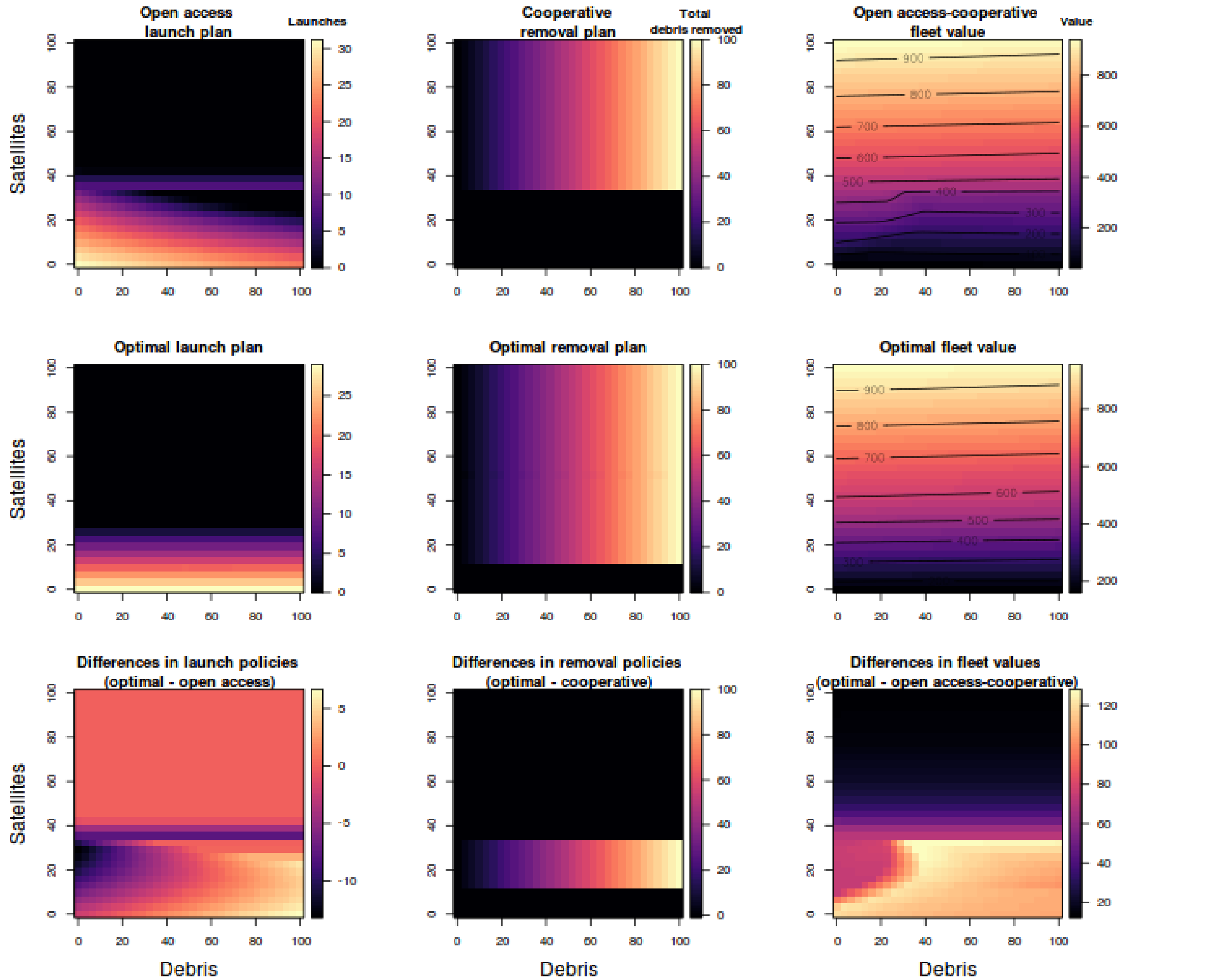

Figure 10: *Comparing optimal and open access-cooperative launch and removal plans.*
Upper row: The open access launch plan (left), cooperative removal plan (middle), and resulting fleet value (right). The jump in the launch plan just above 10 reflects open access launches taking advantage of debris removal beginning, as shown in the time paths in Figure 9.
Middle row: The optimal launch plan (left), optimal removal plan(middle), and resulting fleet value (right).
Bottom row: The gap between optimal plans/values and open access-cooperative plans/values. The gap between optimal and open access-cooperative fleet values is maximized when (a) the planner would begin removing debris but cooperative satellite owners have not, and (b) just before open access launchers begin to launch again (anticipating removal) and the planner has stopped.

Figures in the appendices

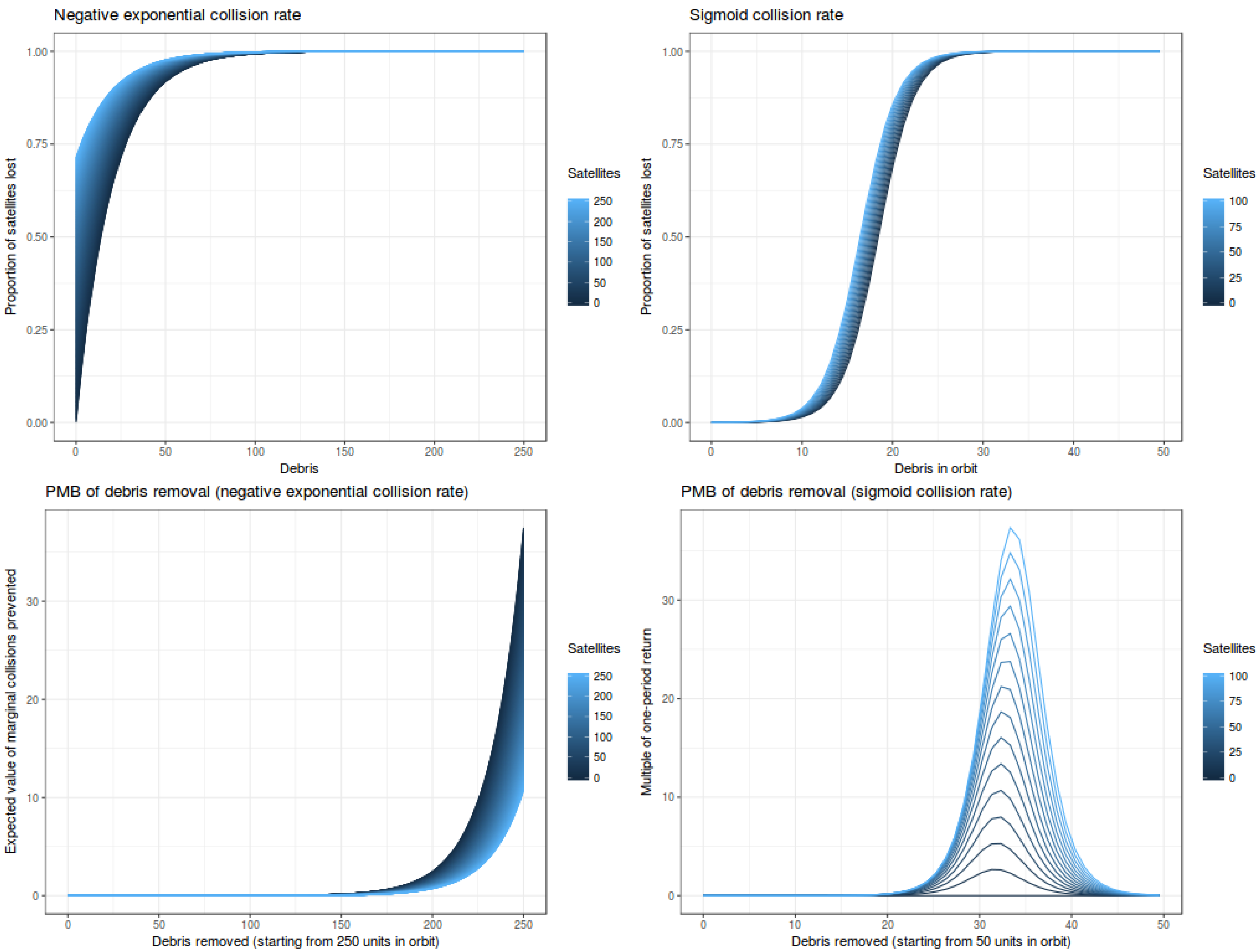

Figure 11: *Two collision rate functions and the private marginal benefit of debris removal.*
Upper row: Collision risks given different levels of debris removal.
Lower row: Private marginal benefits of debris removal.
Left column: Negative exponential collision rate (globally concave).
Right column: Sigmoid collision rate (convex then concave).
Darker colors correspond to fewer satellites. More satellites may reduce or increase the marginal benefits of debris removal, depending on whether satellites and debris are complements or substitutes in collision production.
Not shown: More initial debris in orbit shifts the removal benefit curves to the right. This makes the optimal removal amount increase until a jump to zero removal.

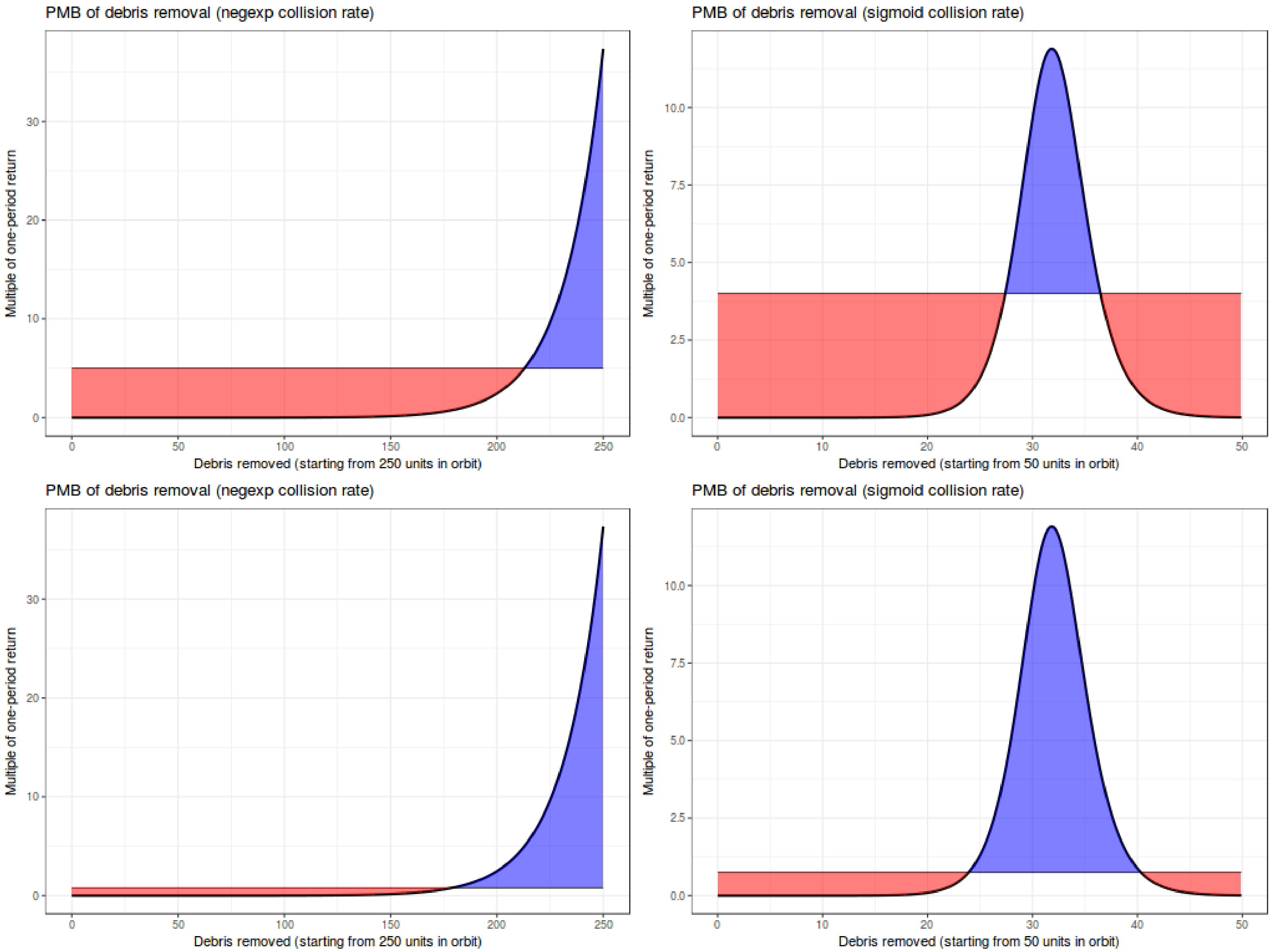

Figure 12: *Nonconvexity and privately optimal removal.*
Upper row: High cost scenario where zero removal is cooperatively optimal.
Lower row: Low cost scenario where some removal is cooperatively optimal.
Left column: Negative exponential collision rate (globally concave), where the optimal removal demand is always on a corner.
Right column: Sigmoid collision rate (convex then concave), where the optimal removal demand may be in the interior.
The thin horizontal line is the marginal cost of removal. The thicker curve is the marginal benefit of removal. Red regions are losses, blue regions are profits. In the upper row, zero removal is optimal. In the lower left panel, full removal is optimal. In the lower right panel, removal of about 40 units is optimal. Because the collision risk is bounded in $[0,1]$, it cannot be strictly convex globally over S and D.

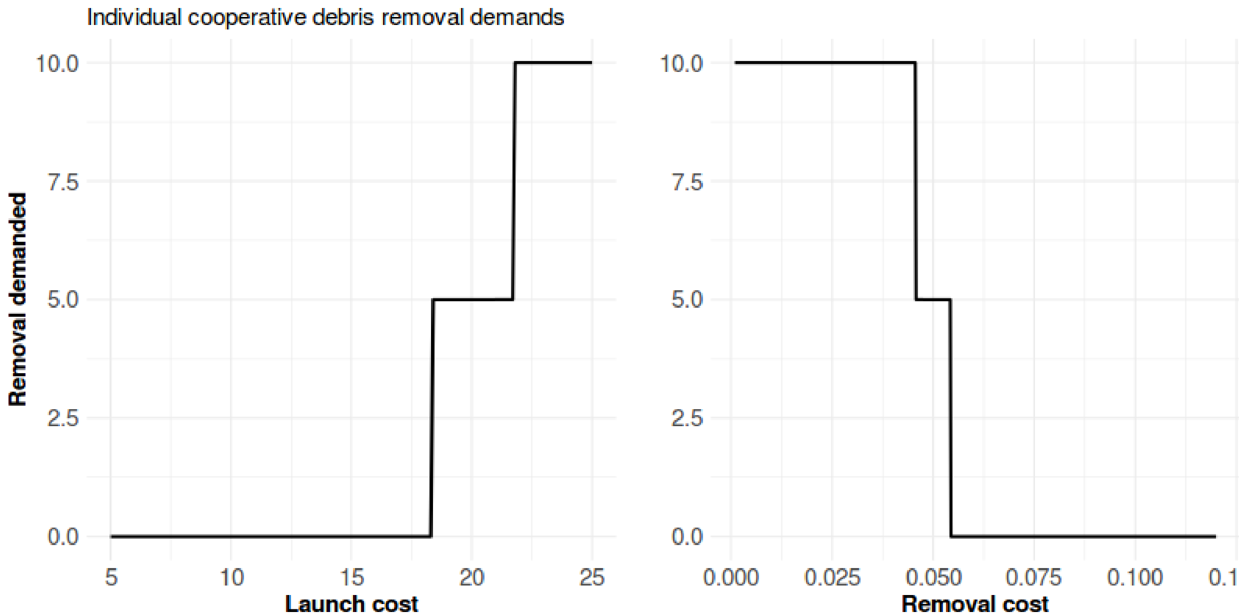

Figure 13: *The effects of changes in satellite launch and debris removal costs on individual cooperative debris removal demands.*
Increases in the cost of launching a satellite increase the open-access value of satellites in orbit, increasing the amount of debris removal demanded. As expected, increases in the cost of debris removal decrease the amount demanded. Costs are stated in multiples of the one-period return generated by a satellite in orbit.

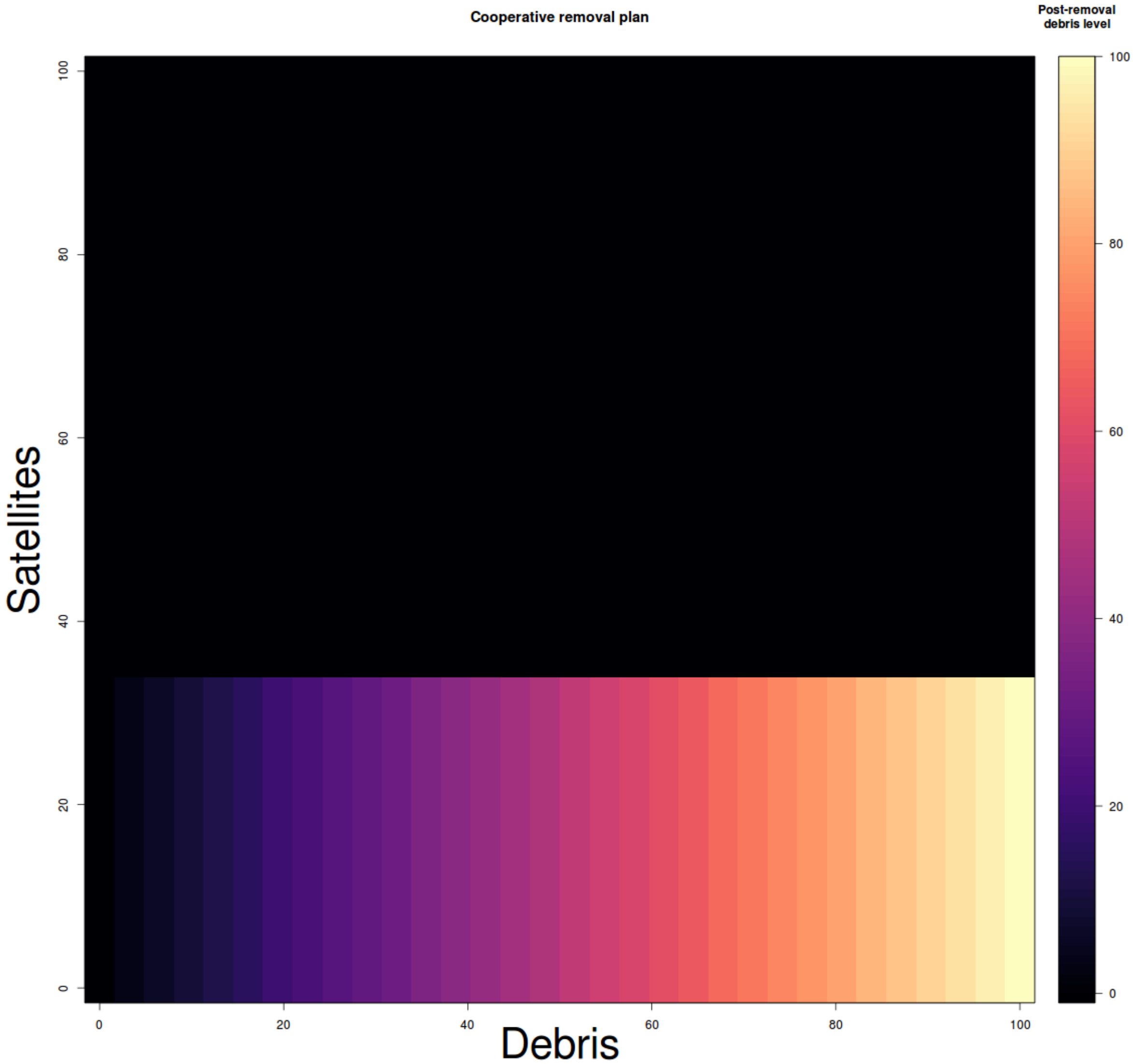

Figure 14: *The effects of changes in the number of firms and debris in orbit on the post-removal level of debris.*
The color scale represents the amount of debris left in orbit after removal. The cooperatively optimal post-removal level of debris does not depend on the amount of debris initially in orbit, but on the number of firms who are available to share the cost of removal. Once there are enough firms to begin removal the post-removal debris level is constant (full removal).

Supplemental appendices can be found here.